

THE PUBLIC ACCOUNTANTS EXAMINATIONS BOARD

A Committee of the Council of ICPAU

CPA (U) EXAMINATIONS

LEVEL ONE

QUANTITATIVE TECHNIQUES - PAPER 5

FRIDAY, 17 JUNE 2011

INSTRUCTIONS TO CANDIDATES

1. Time allowed: **3 hours 15 minutes**.

The first 15 minutes of this examination have been designated for reading time. You may not start to write your answer during this time.

2. Attempt **three** questions in Section **A** and **two** questions in Section **B**
3. Section **A** has **four** questions and only **three** are to be attempted. Each question carries 20 marks.
4. Section **B** has **three** questions and only **two** are to be attempted. Each question carries 20 marks
5. Formulae and tables are provided on pages 7 - 9.
6. Please read further instructions on the answer booklet, before attempting any question.

SECTION A**Question 1**

- (a) Explain the following terms as applied in statistics:
- (i) Central tendency. **(2 marks)**
 - (ii) Variation. **(2 marks)**
- (b) Distinguish between a:
- (i) small sample and a large sample. **(2 marks)**
 - (ii) statistic and a parameter. **(2 marks)**
- (c) The time spent by patients in a doctor's office before receiving medical attention directly affects the efficient operation of the doctor's practice. The number of minutes (after their scheduled appointment time) that each of 56 patients had to wait before seeing an obstetrician at a clinic are as shown below:

| Waiting time (Minutes) | No. of patients |
|------------------------|-----------------|
| 0 – 5 | 15 |
| 6 – 11 | 12 |
| 12 – 17 | 11 |
| 18 – 23 | 10 |
| 24 – 29 | 8 |

Required:

Find the:

- (i) sample variance. **(10 marks)**
 - (ii) Sample standard deviation . **(2 marks)**
- (Total 20 marks)**

Question 2

- (a) Define the following terms as applied in probability theory:
- (i) Null event. **(2 marks)**
 - (ii) Definite event. **(2 marks)**
 - (iii) Mutually exclusive events. **(2 marks)**
- (b) A certain professor taught Quantitative Techniques and Business Management courses. The grade distribution per course is as shown below:

| Grade | Course | |
|-------|-------------------------|---------------------|
| | Quantitative Techniques | Business Management |
| A | 7 | 8 |
| B | 9 | 10 |
| C | 11 | 12 |
| D | 6 | 9 |
| O | 5 | 8 |

A student that was in the Professor's class during the semester is randomly selected.

Required:

Find the probability that the student:

- (i) received grade A. (2marks)
- (ii) was in the Quantitative Techniques class. (2marks)
- (iii) was in the Quantitative Techniques class and received grade A (1 mark)

- (c) (i) List **four** characteristics of a normal distribution curve. (4 marks)
- (ii) A tyre manufacturing company claims that the useful life of its tyres is normally distributed with mean life of 28,000 kilometres and standard deviation of 4,000 kilometres.

Required:

Find the percentage of the tyres that are expected to last more than 35,000 kilometres.

(5 marks)

(Total 20 marks)

Questions 3

- (a) (i) Define the term 'forecasting'. (2 marks)
- (ii) State **three** types of forecasts. (3 marks)
- (b) A researcher on political affairs wishes to determine whether there is a significant correlation between the number of agents employed and the number of votes obtained by a candidate. The table below shows data collected from 12 different polling centres selected at random.

| No. of agents (y) | No. of votes (x) |
|-------------------|------------------|
| 23 | 140 |
| 11 | 101 |
| 10 | 43 |
| 4 | 55 |
| 20 | 79 |
| 14 | 134 |
| 7 | 75 |
| 42 | 211 |
| 3 | 78 |
| 2 | 36 |
| 15 | 45 |
| 6 | 11 |

Required:

- (i) Compute Spearman's rank correlation coefficient. (8 marks)
 (ii) Test the level of significance at 5%. (7 marks)

Total 20 marks)**Question 4**

- (a) Differentiate between interpolation and extrapolation. (4 marks)
 (b) The following information indicates a linear relationship between the values of x and values of y:

| | | | | | |
|---|---|----|----|----|----|
| x | 0 | 6 | 10 | 15 | 20 |
| y | 0 | 13 | 25 | 39 | 56 |

Required:

Use linear interpolation to find the value of:

- (i) y when x = 12 (2 marks)
 (ii) x when y = 14 (2 marks)
 (c) A company keeps records of its monthly expenditure for advertising and its total monthly sales. For the first 10 months in 2010, the records showed the following.

| | | | | | | | | | | |
|--------------------------------|----|----|----|----|----|----|----|----|----|----|
| Advertising cost (Shs million) | 43 | 44 | 36 | 38 | 47 | 40 | 41 | 54 | 37 | 46 |
| Sales (Shs million) | 74 | 76 | 60 | 68 | 79 | 70 | 71 | 94 | 65 | 78 |

Required:

- (i) Find the least squares regression equation for the data. (10 marks)
 (ii) Use the regression equation in (c) (i) above to predict the sales if the company plans to spend Shs 50 million for advertising in the following month, assuming that other factors can be neglected.

(2 marks)**(Total 20 marks)****SECTION B****Question 5**

- (a) Define the terms:
 (i) Time series. (2 marks)
 (ii) Trend. (2 marks)
 (iii) Seasonality. (2 marks)

- (b) The number of barrels of oil imported from the Organisation of Petroleum Exporting Countries (OPEC) by Uganda for the period 1981 to 1984 is as follows:

| Year | No. of barrels |
|------|----------------|
| 1981 | 1,849,017 |
| 1982 | 2,260,482 |
| 1983 | 2,057,468 |
| 1984 | 2,023,341 |

Required:

- (i) Use 1981 as a base year to compute the import index for each of the years. **(6 marks)**
- (ii) Interpret the results in (b) (i) above. **(3 marks)**
- (c) Mrs Okot has discovered that the amount of time she takes to drive to work is normally distributed with a mean of 35 minutes and a standard deviation of 7 minutes.

Required:

Find at what time Mrs. Okot should leave her home so that she has a 95% chance of arriving at work by 9:00a.m.

(5 marks)

(Total 20 marks)

Question 6

- (a) Define the terms:
- (i) Objective function. **(2 marks)**
 - (ii) Shadow price. **(2 marks)**
 - (iii) Feasible region. **(2 marks)**
 - (iv) optimal solution. **(2 marks)**
- (b) Maximise the function $T = 2x + 3y$ subject to the conditions:
- $$x + 4y \geq 8$$
- $$y - 2x \leq 2$$
- $$2x + y \leq 7$$
- $$x, y \geq 0$$
- where x and y are integers.

(6 marks)

- (c) The demand and total cost functions of a firm are given by:

Demand function: $q = 100 - 0.2p$

Total cost function: $TC = 50 + 20q + q^2$.

Required:

Find the:

- (i) price function. (1 mark)
- (ii) total revenue function. (1 mark)
- (iii) profit maximizing output. (4 marks)

(Total 20 marks)

Question 7

- (a) Define the terms:
 - (i) Gantt chart. (2 marks)
 - (ii) Network. (2 marks)
- (b)
 - (i) Define the word quality. (2 marks)
 - (ii) State four types of control charts. (4 marks)
- (c) A project consists of five activities A, B, C, D and E that satisfy the following precedence relationships:
 1. Neither A nor B has any immediate predecessors.
 2. A is an immediate predecessor of C.
 3. B is an immediate predecessor of D.
 4. C and D are immediate predecessors of E.

Required:

Draw a network for this project. (5 marks)

- (d) A truck manufacturing company claims that the average weight of its loaded trucks is 11,000 kg with standard deviation of 800 kg. A highway traffic police inspector decided to check this claim. He randomly checked 36 trucks and found that the average weight of the loaded trucks is 12,500 kg.

Required:

Test at a 5% level of significance, the claim that the average weight of the loaded trucks is more than 11,000 kg.

(5 marks)

(Total 20 marks)

FORMULAE

1. **Spearman's rank correlation coefficient** $r_s = 1 - \frac{6\sum d^2}{n^3 - n}$

Where r_s = Spearman's Rank correlation Coefficient

n = Number of observations

d = the difference of ranks between paired items in the two series.

2. **The formula to obtain b and a are:**

$$b = \frac{n(\sum XY) - (\sum X)(\sum Y)}{n(\sum X^2) - (\sum X)^2}$$

$$a = \frac{\sum Y}{n} - b \frac{\sum X}{n}$$

Where X is the independent variable

Y is the dependent variable

n is the number of items in the sample.

3. **Standard normal distribution** $z = \frac{x - \mu}{\delta}$

4. **Variance** $s^2 = \frac{n(\sum fx^2) - (\sum fx)^2}{n(n-1)}$

5. **Level of significance** $t = r_s \sqrt{\frac{n-2}{1-r_s^2}}$

| NORMAL DISTRIBUTION $N(0,1)$ $\phi(Z)$ | | | | | | | | | | | SUBTRACT | | | | | | | | |
|--|--------|------|------|------|------|------|------|------|------|------|----------|---|---|---|---|---|----|----|----|
| Z | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 0.0 | 0.3989 | 3989 | 3989 | 3988 | 3986 | | | | | | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 3 |
| 0.1 | 0.3970 | 3965 | 3961 | 3956 | 3951 | 3984 | 3982 | 3980 | 3977 | 3973 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 0.2 | 0.3910 | 3902 | 3894 | 3885 | 3876 | 3945 | 3939 | 3932 | 3925 | 3918 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 6 |
| 0.3 | 0.3814 | 3802 | 3790 | 3778 | 3765 | 3867 | 3857 | 3847 | 3836 | 3825 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 |
| 0.4 | 0.3683 | 3668 | 3653 | 3637 | 3621 | 3752 | 3739 | 3725 | 3712 | 3697 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 10 |
| 0.5 | 0.3521 | 3503 | 3485 | 3467 | 3448 | 3605 | 3589 | 3572 | 3555 | 3538 | 1 | 3 | 4 | 5 | 6 | 7 | 8 | 10 | 11 |
| 0.6 | 0.3332 | 3312 | 3292 | 3271 | 3251 | 3725 | 3709 | 3692 | 3675 | 3657 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 10 | 11 |
| 0.7 | 0.3123 | 3101 | 3079 | 3056 | 3034 | 3605 | 3589 | 3572 | 3555 | 3538 | 2 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 0.8 | 0.2897 | 2874 | 2850 | 2827 | 2803 | 3034 | 3011 | 2989 | 2966 | 2943 | 2 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 0.9 | 0.2661 | 2637 | 2613 | 2589 | 2565 | 2780 | 2756 | 2732 | 2709 | 2685 | 2 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 1.0 | 0.2420 | 2396 | 2371 | 2347 | 2323 | 2516 | 2492 | 2468 | 2444 | 2420 | 2 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 1.1 | 0.2179 | 2155 | 2131 | 2107 | 2083 | 2299 | 2275 | 2251 | 2227 | 2203 | 2 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 1.2 | 0.1942 | 1919 | 1895 | 1872 | 1849 | 2059 | 2036 | 2012 | 1989 | 1965 | 2 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 1.3 | 0.1714 | 1691 | 1669 | 1647 | 1626 | 1826 | 1804 | 1781 | 1758 | 1736 | 2 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 1.4 | 0.1497 | 1476 | 1456 | 1435 | 1415 | 1582 | 1561 | 1539 | 1518 | 1497 | 2 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 1.5 | 0.1295 | 1276 | 1257 | 1238 | 1219 | 1394 | 1374 | 1354 | 1334 | 1315 | 2 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 1.6 | 0.1109 | 1092 | 1074 | 1057 | 1040 | 1200 | 1182 | 1163 | 1145 | 1127 | 2 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 1.7 | 0.0940 | 0925 | 0909 | 0893 | 0878 | 1006 | 0989 | 0973 | 0957 | 0940 | 2 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 1.8 | 0.0790 | 0775 | 0761 | 0748 | 0734 | 0863 | 0848 | 0833 | 0818 | 0804 | 2 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 1.9 | 0.0656 | 0644 | 0632 | 0620 | 0608 | 0721 | 0707 | 0694 | 0681 | 0669 | 2 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 2.0 | 0.0540 | 0529 | 0519 | 0508 | 0498 | 0596 | 0584 | 0573 | 0562 | 0551 | 2 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 2.1 | 0.0440 | 0431 | 0422 | 0413 | 0404 | 0488 | 0478 | 0468 | 0459 | 0449 | 2 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 2.2 | 0.0355 | 0347 | 0339 | 0332 | 0325 | 0396 | 0387 | 0379 | 0371 | 0363 | 2 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 2.3 | 0.0283 | 0277 | 0270 | 0264 | 0258 | 0317 | 0310 | 0303 | 0297 | 0290 | 2 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 2.4 | 0.0224 | 0219 | 0213 | 0208 | 0203 | 0252 | 0246 | 0241 | 0235 | 0229 | 2 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 2.5 | 0.0175 | 0171 | 0167 | 0163 | 0158 | 0198 | 0194 | 0189 | 0184 | 0180 | 2 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 2.6 | 0.0136 | 0132 | 0129 | 0126 | 0122 | 0154 | 0151 | 0147 | 0143 | 0139 | 2 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 2.7 | 0.0104 | 0101 | 0099 | 0096 | 0093 | 0119 | 0116 | 0113 | 0110 | 0107 | 2 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 2.8 | 0.0079 | 0077 | 0075 | 0073 | 0071 | 0069 | 0067 | 0065 | 0063 | 0061 | 2 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 2.9 | 0.0060 | 0058 | 0056 | 0055 | 0053 | 0051 | 0050 | 0048 | 0047 | 0046 | 2 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 3.0 | 0.0044 | 0033 | 0024 | 0017 | 0012 | 0009 | 0006 | 0004 | 0003 | 0002 | 2 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |

The functions tabled are:

$$\phi(Z) = \sqrt{\frac{1}{2\pi}} \exp\left(-\frac{1}{2}Z^2\right), \text{ where } \phi(Z) \text{ is the probability density of the standardized normal distribution } N(0,1)$$

CRITICAL POINTS OF THE NORMAL DISTRIBUTION Z_p

| P | Q | z | P | Q | z | P | Q | z |
|------|------|-------|------|------|-------|---------|---------|-------|
| .00 | .50 | 0.000 | .460 | .040 | 1.751 | .490 | .010 | 2.326 |
| .05 | .45 | 0.126 | .462 | .038 | 1.774 | .491 | .009 | 2.366 |
| .10 | .40 | 0.253 | .464 | .036 | 1.799 | .492 | .008 | 2.409 |
| .15 | .35 | 0.385 | .466 | .034 | 1.825 | .493 | .007 | 2.457 |
| .20 | .30 | 0.524 | .468 | .032 | 1.852 | .494 | .006 | 2.512 |
| .25 | .25 | 0.674 | .470 | .030 | 1.881 | .495 | .005 | 2.576 |
| .30 | .20 | 0.842 | .472 | .028 | 1.911 | .496 | .004 | 2.652 |
| .35 | .15 | 1.036 | .474 | .026 | 1.943 | .497 | .003 | 2.748 |
| .40 | .10 | 1.282 | .476 | .024 | 1.977 | .498 | .002 | 2.878 |
| .45 | .05 | 1.645 | .478 | .022 | 2.014 | .499 | .001 | 3.090 |
| .450 | .050 | 1.645 | .480 | .020 | 2.054 | .4995 | .0005 | 3.291 |
| .452 | .048 | 1.665 | .482 | .018 | 2.097 | .4999 | .0001 | 3.719 |
| .454 | .046 | 1.685 | .484 | .016 | 2.144 | .49995 | .00005 | 3.891 |
| .456 | .044 | 1.706 | .486 | .014 | 2.197 | .49999 | .00001 | 4.265 |
| .458 | .042 | 1.728 | .488 | .012 | 2.257 | .499995 | .000005 | 4.417 |

PERCENTAGE POINTS OF STUDENT'S t -DISTRIBUTION t_Q

| ν | Probability* | | | | | | | | | Q |
|----------|--------------|--------------|--------------|----------------|--------------|----------------|------------------|----------------|------------------|-------|
| | 0.25 0.50 | 0.10 0.20 | 0.05 0.10 | 0.025 0.050 | 0.01 0.02 | 0.005 0.010 | 0.0025 0.0050 | 0.001 0.002 | 0.0005 0.0010 | |
| 1 | 1.000 | 3.078 | 6.314 | 12.71 | 31.82 | 63.66 | 127.3 | 318.3 | 636.6 | |
| 2 | 0.816 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 14.09 | 22.33 | 31.60 | |
| 3 | 0.765 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 7.453 | 10.21 | 12.92 | |
| 4 | 0.741 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 5.598 | 7.173 | 8.610 | |
| 5 | 0.727 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 4.773 | 5.893 | 6.869 | |
| 6 | 0.718 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 4.317 | 5.208 | 5.959 | |
| 7 | 0.711 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 4.029 | 4.785 | 5.408 | |
| 8 | 0.706 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 3.833 | 4.501 | 5.041 | |
| 9 | 0.703 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 3.690 | 4.297 | 4.781 | |
| 10 | 0.700 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 3.581 | 4.144 | 4.587 | |
| 11 | 0.697 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 3.497 | 4.025 | 4.437 | |
| 12 | 0.695 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 3.428 | 3.930 | 4.318 | |
| 13 | 0.694 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 3.372 | 3.852 | 4.221 | |
| 14 | 0.692 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 3.326 | 3.787 | 4.140 | |
| 15 | 0.691 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 3.286 | 3.733 | 4.073 | |
| 16 | 0.690 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 3.252 | 3.686 | 4.015 | |
| 17 | 0.689 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 3.222 | 3.646 | 3.965 | |
| 18 | 0.688 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 3.197 | 3.610 | 3.922 | |
| 19 | 0.688 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 3.174 | 3.579 | 3.883 | |
| 20 | 0.687 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 3.153 | 3.552 | 3.850 | |
| 21 | 0.686 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.135 | 3.527 | 3.819 | |
| 22 | 0.686 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 3.119 | 3.505 | 3.792 | |
| 23 | 0.685 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 3.104 | 3.485 | 3.767 | |
| 24 | 0.685 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 3.091 | 3.467 | 3.745 | |
| 25 | 0.684 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 3.078 | 3.450 | 3.725 | |
| 26 | 0.684 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 3.067 | 3.435 | 3.707 | |
| 27 | 0.684 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 3.057 | 3.421 | 3.690 | 120 |
| 28 | 0.683 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 3.047 | 3.408 | 3.674 | ν |
| 29 | 0.683 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 3.038 | 3.396 | 3.659 | |
| 30 | 0.683 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 3.030 | 3.385 | 3.646 | 4 |
| 40 | 0.681 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 2.971 | 3.307 | 3.551 | 3 |
| 60 | 0.679 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 | 2.915 | 3.232 | 3.460 | 2 |
| 120 | 0.677 | 1.289 | 1.658 | 1.980 | 2.358 | 2.617 | 2.860 | 3.160 | 3.373 | 1 |
| ∞ | 0.674 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 2.807 | 3.090 | 3.291 | 0 |

The function tabulated is t_Q defined by

$$\int_{t_Q}^{\infty} f(t) dt = Q; \quad f(t) = \frac{(\frac{1}{2}\nu - \frac{1}{2})!}{\sqrt{(v\pi)(\frac{1}{2}\nu - 1)!}} \cdot \frac{1}{(1 + t^2/\nu)^{(\nu+1)/2}}$$

where $f(t)$ is the probability density of the t -distribution.

Interpolation ν -wise should be linear in $120/\nu$ for $\nu > 30$.

Use (i) upper row for one tail-tests

(i) lower row for two tail-tests

If x is a random variable with the t -probability distribution for ν degrees of freedom, the probability that $x > t_Q$ is Q and the probability that $|x| > t_Q$ is $2Q$.

The graph shows the form of the distribution for $\nu = 2$. The shaded area represents the probability Q . For large ν the distribution approximates to the normal distribution $N(0,1)$, shown by the dotted line.

