THE PUBLIC ACCOUNTANTS EXAMINATIONS BOARD

A Committee of the Council of ICPAU

CPA (U) EXAMINATIONS

LEVEL ONE

QUANTITATIVE TECHNIQUES - PAPER 2

TUESDAY 27 NOVEMBER, 2018

INSTRUCTIONS TO CANDIDATES

1. Time allowed: **3 hours 15 minutes**.
   The first 15 minutes of this examination have been designated for reading time. You may not start to write your answer during this time.
2. This examination contains **six** questions and only **five** questions are to be attempted. Each question carries 20 marks.
3. Formulae and tables are provided on pages 8 – 12.
4. Write your answer to each question on a fresh page in your answer booklet.
5. Please, read further instructions on the answer booklet, before attempting any question.

Attempt five of the six questions

Question 1

(a) Explain the term 'tables' in terms of summarising raw data. **(2 marks)**

(b) A dairy sold milk, in liters, on a daily basis for a period of 24 days as shown in the table below:

28	36	41	22	44	23
24	45	20	26	53	54
57	43	21	29	60	42
33	28	39	44	49	52

Required:

(i) Using a uniform class interval and starting with the class 20-24; construct a frequency distribution for the data above. **(2 marks)**

(ii) Calculate the standard deviation for the data. **(7 marks)**

(c) A beverages company produced and sold crates of dry gin in the years 2016 and 2017 as shown in the following table.

Month	2016	2017
January	20	30
February	50	40
March	30	50
April	40	40
May	60	50
June	100	90
July	70	110
August	90	120
September	60	80
October	80	100
November	50	70
December	120	110

Required:

(i) Construct a Z-chart for the above data. **(8 marks)**

(ii) Comment on any feature in the Z-chart you have drawn. **(1 mark)**

(Total 20 marks)

Question 2

Mongoli Enterprises Ltd (NEL) deals in solar products. A routine visit by Uganda National Bureau of Standards (UNBS) established that the lifetime of the solar panels NEL sold was normally distributed with a mean of 25 years and standard deviation of 4 years. A quality controller from UNBS later visited NEL, tested a sample of solar panels and picked one at random.

Required:

Determine the:

(i) probability that its lifetime is greater than 18 years. **(3 marks)**

(ii) probability that its lifetime is between 18 and 28 years. **(3 marks)**

(iii) sample size of the panels that were tested if only two failed to exceed a life time of 18 years. **(3 marks)**

(b) Zakaria, a teller at Eurit Bank handles clients with bulk transactions. On a certain day, he examined a random sample of 9 transactions and noted that the mean and variance of the sample were Shs 15.2 million and Shs 5 million respectively.

Required:

Determine the range of the average transaction at 95% confidence level. **(4 marks)**

(c) Tuge Ltd sells on average 3 motorcycles daily and the sales are believed to follow a Poisson distribution.

Required:

Find the probability that the motorcycle sales are:

(i) exactly two. **(2 marks)**

(ii) at most one. **(3 marks)**

(iii) at least two. **(2 marks)**

(Total 20 marks)

Question 3

(a) Explain any **one** application of a regression line. **(1 mark)**

(b) The maize yield on a farm is believed to depend on the amount of rainfall. The values of the yield Q (tons per acre) and the rainfall P (in mm) for seven successive seasons are given in the table below.

P	125	137	145	112	132	141	120
Q	6.25	8.02	8.42	5.3	7.2	8.7	5.7

Required:

(i) Compute the product-moment correlation coefficient between rainfall and yield.

(9 marks)

(ii) Comment on the result obtained in (b) (i) above. **(1 mark)**

(c) A sample of six households were interviewed regarding their incomes (x) and expenditure (y). The results (in Shs '000') are as shown below:

x	230	340	370	380	140	280
y	80	90	160	180	50	70

Required:

(i) Determine the regression equation relating y to x. **(8 marks)**

(ii) Find the income of a household with an expenditure of Shs 60,000.

(1 mark)

(Total 20 marks)

Question 4

(a) A roofing materials company produces ridges (x) and iron sheets (y) using resources according to the production function given by: $x + y^2 + 4y = 20$.

Required:

Determine the number of ridges and iron sheets produced given that for every 4 iron sheets, 1 ridge is produced.

(5 marks)

(b) A famous writer of project planning textbooks would like to be paid a royalty of 15% of the sales revenue and insists the price should maximise total revenue. On the other hand the publishing company is interested in maximising profit. Given the total revenue and total cost functions are $R(q) = 10{,}000q - 10q^2$ and $C(q) = 1{,}000 + 2{,}500q + 5q^2$ respectively, where q is the number of textbooks sold.

Required:

Determine the:

(i) output that maximises total revenue and obtain the maximum royalty.

(5 marks)

(ii) output that maximises profit and associated royalty at that output.

(6 marks)

(c) The approximate sales, in thousands of shillings, for a small retail shop in a given week are given in the probability distribution table below:

Sales (x)	100	105	110	115	120
Probability (p)	0.01	0.08	0.20	0.50	0.21

Required:

Calculate the variance for the distribution. **(4 marks)**

Question 5

(a) Explain any **two** limitations of linear programming. **(2 marks)**

(b) Airplanes from YK airport sometimes delay to take off. The quality control manager decided to randomly choose 5 flights per day for eight days. The table below shows the number of minutes that the planes were late for takeoff.

Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8
17	5	12	4	3	10	2	3
3	14	5	21	9	22	14	5
22	4	6	8	15	40	13	23
6	13	8	5	35	26	8	33
24	14	4	19	5	23	12	23

Required:

(i) Draw a control chart for the range for the above information. **(8 marks)**

(ii) Given that the coefficients for the upper control and the lower control limits are 2.114 and 0 respectively; determine whether the process is out of control. **(1 mark)**

(c) An employer wants to recruit experienced workers (x) and those on probation (y) for his firm. The firm can accommodate at most 9 workers. On average, 5 hours and 3 hours of work are required of the experienced workers and those on probation respectively. The employer has to maintain an output of at least 30 hours of work per day. The rules and regulations demand that the number of experienced workers recruited should not be more than 5 times the number of those on probation. The workers union, however, emphasizes that the number of experienced workers recruited should be at least twice the number of those on probation.

Required:

(i) Formulate linear inequalities from the information given. **(4 marks)**

(ii) Illustrate the above information on a graph. **(4 marks)**

(iii) Obtain all the possible combinations of workers that may be recruited. **(1 mark)**

(Total 20 marks)

Question 6

(a) Distinguish between normal costs and crash costs under network analysis. **(2 marks)**

(b) KA Construction Ltd is in the process of designing an improved drainage system in one of the suburbs of Kampala. The activities and duration times required are listed in precedence table below:

Activity	Preceding activity	Duration (days)
A	-	6
B	A	8
C	A	7
D	C	12
E	B	3
F	D	5
G	E, F	7

Required:

(i) Draw an activity network. **(2 marks)**

(ii) List the possible paths through the network and their duration. **(4 marks)**

(iii) Identify the critical path and justify your answer. **(2 marks)**

(c) (i) Distinguish between the fixed base and chain base methods of computing index numbers. **(2 marks)**

(ii) Explain **three** limitations of consumer price index numbers. **(3 marks)**

(iii) The table below shows the average prices (in Shs '000') for a ton of each of the following commodities in 2016 and 2017.

Commodities	Prices in 2016	Prices in 2017
Maize	600	700
Beans	850	900
Sorghum	520	500
Millet	550	620

Required:

Calculate the simple aggregate price index for 2017 using 2016 as the base year and comment on your results. **(5 marks)**

(Total 20 marks)

FORMULAE

1. Combination $^{n}C_{r} = \frac{n!}{(n-r)!r!}$
2. Permutations $^{n}p_{r} = \frac{n!}{(n-r)!}$
3. Mean of the binomial distribution $= np$
4. Standard deviation $= \sqrt{npq}$
5. Variance of the binomial distribution $= np(1-p)$
6. Standard error of population proportion $S_{ps} = \sqrt{\frac{pq}{n}}$
7. Spearman's rank correlation coefficient $r = 1 - \frac{6\sum d^2}{n(n^2-1)}$
8. Product moment coefficient of correlation $= \frac{n\sum xy - \sum x \sum y}{\sqrt{(n\sum x^2 - (\sum x)^2) \times (n\sum y^2 - (\sum y)^2)}}$
9. Cost slope $= \frac{\text{crash cost – normal cost}}{\text{normal time – crash time}}$
10. Harmonic mean (ungrouped data) $hm = \frac{n}{\sum \frac{1}{x}}$
11. Sample mean $\bar{x} = \frac{\sum x}{n}$
12. Harmonic mean (grouped data) $hm = \frac{n}{\sum \frac{f}{x}}$
13. Quartile coefficient of dispersion $= \frac{Q_3 - Q_1}{Q_3 + Q_1}$
14. Mean $\bar{x} = A + \frac{\sum fd}{\sum f}$ or Mean $\bar{x} = \frac{\sum fx}{\sum f}$
15. Median $= Lb + \left(\frac{\frac{N}{2} - Cfb}{fm} \right) C$
16. Mode $= lm + \left(\frac{d_1}{d_1 + d_2} \right) C$

17. Variance $Var(x)=\frac{\sum fx^2}{\sum f}-\left(\frac{\sum fx}{\sum f}\right)^2$

18. Standard deviation $\delta=\sqrt{\frac{\sum fx^2}{\sum f}-\bar{x}^2} \quad =\sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}}$

19. Sample standard deviation $s=\sqrt{\frac{\sum (x-\bar{x})^2}{n-1}}$

20. Least squares regression equation of y on x is given by; $y=a+bx$
    Where; $b=\frac{n\sum xy-\sum x\sum y}{n\sum x^2-\left(\sum x\right)^2}$ and $a=\frac{\sum y}{n}-\frac{b\sum x}{n}$

21. Least squares regression equation of x on y is given by; x = c +dy
    Where $c=\frac{\sum x}{n}-\frac{d\sum y}{n}$ and $d=\frac{n\sum xy-\sum x\sum y}{n\sum y^2-(\sum y)^2}$

22. Standardizing normal. $z=\frac{\bar{x}-\mu}{\sigma}$

23. Confidence interval for sample mean $=\bar{x}\pm t_{\alpha/2}\frac{s}{\sqrt{n}}$

24. $\chi^2=\sum\frac{(O-E)^2}{E}$

25. Confidence interval of proportion $= p\pm z_{\alpha/2}\sqrt{\frac{pq}{n}}$

26. Pearson coefficient of skewness $Sk=\frac{(\bar{x}-\text{mode})}{s_d}$ or $Sk=\frac{3\left(\bar{x}-median\right)}{s_d}$

27. Expectation $=\sum xP(X=x)$

28. Laspeyres' price index $=\frac{\sum (p_1\times q_0)}{\sum (q_0\times p_0)}\times 100$

29. Weighted aggregate price index $=\frac{\sum wv_n}{\sum wv_0}\times 100$

30. Additive law of probability; $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

FORMULAE

31.	Conditional probability $P\left(A/B\right)=\frac{P(A\cap B)}{P(B)}$
32.	Independence of A, B $P\left(A/B\right)=P(A) or P(A\cap B)=P(A)\times P(B)$
33.	Continuous compounding $A=P(1+r)^n+\frac{b(1+r)^n-b}{r}$
34.	Quotient rule of differentiation $f=\frac{vu^1-uv^1}{v^2}$; where $f=\frac{u}{v}$
35.	$Paasche's Model=\frac{\sum(p_1\times q_1)}{\sum(q_1\times p_0)}\times 100$
36.	$Poisson Model P(X=x)=e^{-\lambda}\frac{\lambda^x}{x!}$

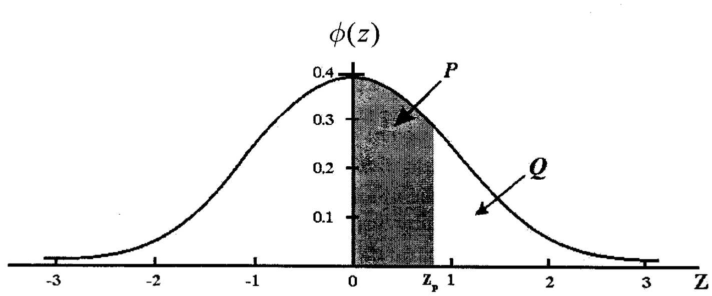
CUMULATIVE NORMAL DISTRIBUTION $P(z)$

											ADD								
Z	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0.0	0.0000	0040	0080	0120	0160	0199	0239	0279	0319	0359	4	8	12	16	20	24	28	32	36
0.1	0.0398	0438	0478	0517	0557	0596	0636	0675	0714	0753	4	8	12	16	20	24	28	32	36
0.2	0.0793	0832	0871	0910	0948	0987	1026	1064	1103	1141	4	8	12	15	19	22	27	31	35
0.3	0.1179	1217	1255	1293	1331	1368	1406	1443	1480	1517	4	8	11	15	19	22	26	30	34
0.4	0.1554	1591	1628	1664	1700	1736	1772	1808	1844	1879	4	7	11	14	18	22	25	29	32
0.5	0.1915	1950	1985	2019	2054	2088	2123	2157	2190	2224	3	7	10	14	17	21	24	27	31
0.6	0.2257	2291	2324	2357	2389	2422	2454	2486	2517	2549	3	6	10	13	16	19	23	26	29
0.7	0.2580	2611	2642	2673							3	6	9	12	15	19	22	25	28
					2704	2734	2764	2794	2823	2852	3	6	9	12	15	18	21	24	27
0.8	0.2881	2910	2939	2967	2995	3023					3	6	8	11	14	17	20	22	25
							3051	3078	3106	3133	3	5	8	11	13	16	19	22	24
0.9	0.3159	3186	3212	3238	3264	3289					3	5	8	10	13	16	18	21	23
							3315	3340	3365	3389	2	5	7	10	12	15	17	20	22
1.0	0.3413	3438	3461	3485	3508						2	5	7	10	12	14	17	19	22
						3531	3554	3577	3599	3621	2	4	7	9	11	13	15	18	20
1.1	0.3643	3665	3686	3708							2	4	6	8	11	13	15	17	19
					3729	3749	3770	3790	3810	3830	2	4	6	8	10	12	14	16	18
1.2	0.3849	3869	3888	3907	3925						2	4	6	8	10	11	13	15	17
						3944	3962	3980	3997	4015	2	4	5	7	9	11	13	14	16
1.3	0.4032	4049	4066	4082	4099	4115	4131	4147	4162	4177	2	3	5	6	8	10	11	13	14
1.4	0.4192	4207	4222	4236	4251	4265	4279	4292	4306	4319	1	3	4	6	7	8	10	11	13
1.5	0.4332	4345	4357	4370	4382	4394	4406	4418	4429	4441	1	2	4	5	6	7	8	10	11
1.6	0.4452	4463	4474	4484	4495	4505	4515	4525	4535	4545	1	2	3	4	5	6	7	8	9
1.7	0.4554	4564	4573	4582	4591	4599	4608	4616	4625	4633	1	2	3	3	4	5	6	7	8
1.8	0.4641	4649	4656	4664	4671	4678	4686	4693	4699	4706	1	1	2	3	4	4	5	6	6
1.9	0.4713	4719	4726	4732	4738	4744	4750	4756	4761	4767	1	1	2	2	3	4	4	5	5
2.0	0.4772	4778	4783	4788	4793	4798	4803	4808	4812	4817	0	1	1	2	2	3	3	4	4
2.1	0.4821	4826	4830	4834	4838	4842	4846	4850	4854	4957	0	1	1	2	2	2	3	3	4
2.2	0.4861	4864	4868	4871	4875	4878	4881	4884	4887	4890	0	1	1	1	2	2	2	3	3
2.3	0.4893	4896	4898	4901	4904	4906	4909	4911	4913	4916	0	0	1	1	1	2	2	2	2
2.4	0.4918	4920	4922	4925	4927	4929	4931	4932	4934	4936	0	0	1	1	1	1	1	2	2
2.5	0.4938	4940	4941	4943	4945	4946	4948	4949	4951	4952									
2.6	0.4953	4955	4956	4957	4959	4960	4961	4962	4963	4964									
2.7	0.4965	4966	4967	4968	4969	4970	4971	4972	4973	4974									
2.8	0.4974	4975	4976	4977	4977	4978	4979	4979	4980	4981									
2.9	0.4981	4982	4982	4983	4984	4984	4985	4985	4986	4986									
3.0	0.4987	4990	4993	4995	4997	4998	4998	4999	4999	5000									

The table gives $P(z) = \int_0^z \phi(z)dz$

If the random variable Z is distributed as the standard normal distribution N(0,1) then:

1. $P(o < Z < z_p) = P(\text{Shaded Area})$
2. $P(Z > Z_p) = Q = ½ - P$
3. $P(Z > |Z_p|) = 1 - 2P = 2Q$



PERCENTAGE POINTS OF THE CHI-SQUARE (χ^2) DISTRIBUTION χ_Q^2

	Probability Q									
ν	**0.995**	**0.990**	**0.975**	**0.950**	**0.100**	**0.050**	**0.025**	**0.010**	**0.005**	**0.001**
1	0.0^4393	0.0^3157	0.0^3982	0.0^2393	2.706	3.841	5.024	6.635	7.879	10.83
2	0.0100	0.0201	0.0506	0.1026	4.605	5.991	7.378	9.210	10.60	13.82
3	0.0717	0.1148	0.2158	0.3518	6.251	7.815	9.348	11.34	12.84	16.27
4	0.2070	0.2971	0.4844	0.7107	7.779	9.488	11.14	13.28	14.86	18.47
5	0.4117	0.5543	0.8312	1.145	9.236	11.07	12.83	15.09	16.75	20.52
6	0.6757	0.8721	1.237	1.635	10.64	12.59	14.45	16.81	18.55	22.46
7	0.9893	1.239	1.690	2.167	12.02	14.07	16.01	18.48	20.28	24.32
8	1.344	1.646	2.180	2.733	13.36	15.51	17.53	20.09	21.95	26.12
9	1.735	2.088	2.700	3.325	14.68	16.92	19.02	21.67	23.59	27.88
10	2.156	2.558	3.247	3.940	15.99	18.31	20.48	23.21	25.19	29.59
11	2.603	3.053	3.816	4.575	17.28	19.68	21.92	24.73	26.76	31.26
12	3.074	3.571	4.404	5.226	18.55	21.03	23.34	26.22	28.30	32.91
13	3.565	4.107	5.009	5.892	19.81	22.36	24.74	27.69	29.82	34.53
14	4.075	4.660	5.629	6.571	21.06	23.68	26.12	29.14	31.32	36.12
15	4.601	5.229	6.262	7.261	22.31	25.00	27.49	30.58	32.80	37.70
16	5.142	5.812	6.908	7.962	23.54	26.30	28.85	32.00	34.27	39.25
17	5.697	6.408	7.564	8.672	24.77	27.59	30.19	33.41	35.72	40.79
18	6.265	7.015	8.231	9.390	25.99	28.87	31.53	34.81	37.16	42.31
19	6.844	7.633	8.907	10.12	27.20	30.14	32.85	36.19	38.58	43.82
20	7.434	8.260	9.591	10.85	28.41	31.41	34.17	37.57	40.00	45.31
21	8.034	8.897	10.28	11.59	29.62	32.67	35.48	38.93	41.40	46.80
22	8.643	9.542	10.98	12.34	30.81	33.92	36.78	40.29	42.80	48.27
23	9.260	10.20	11.69	13.09	32.01	35.17	38.08	41.64	44.18	49.73
24	9.886	10.86	12.40	13.85	33.20	36.42	39.36	42.98	45.56	51.18
25	10.52	11.52	13.12	14.61	34.38	37.65	40.65	44.31	46.93	52.62
26	11.16	12.20	13.84	15.38	35.56	38.89	41.92	45.64	48.29	54.05
27	11.81	12.88	14.57	16.15	36.74	40.11	43.19	46.96	49.64	55.48
28	12.46	13.56	15.31	16.93	37.92	41.34	44.46	48.28	50.99	56.89
29	13.12	14.26	16.05	17.71	39.09	42.56	45.72	49.59	52.34	58.30
30	13.79	14.95	16.79	18.49	40.26	43.77	46.98	50.89	53.67	59.70
40	20.71	22.16	24.43	26.51	51.81	55.76	59.34	63.69	66.77	73.40
50	27.99	29.71	32.36	34.76	63.17	67.50	71.42	76.15	79.49	86.66
60	35.53	37.48	40.48	43.19	74.40	79.08	83.30	88.38	91.95	99.61
70	43.28	45.44	48.76	51.74	85.53	90.53	95.02	100.4	104.2	112.3
80	51.17	53.54	57.15	60.39	96.58	101.9	106.6	112.3	116.3	124.8
90	59.20	61.75	65.65	69.13	107.6	113.1	118.1	124.1	128.3	137.2
100	67.33	70.06	74.22	77.93	118.5	124.3	129.6	135.8	140.2	149.4

The function tabulated is χ_Q^2 defined by

$$\int_{\chi_Q^2}^{\infty} f(x)dx = Q; \quad f(x) = \frac{1}{2^{\nu/2}(\frac{1}{2}\nu - 1)!} x^{\nu/2 - 1} e^{-x/2} \quad (x > 0)$$

where *f(x)* is the probability density of the χ^2 distribution for ν degrees of freedom. Interpolation ν-wise for $\nu > 30$ gives adequate values (but errors up to 5 units in the last figure may occur for the smaller ν). For $\nu > 100$ the distribution of $\sqrt{(2\chi^2)}$ is approximately normal with mean $\sqrt{(2\nu - 1)}$ and unit variance.

Note: $0.0^4 2 = 0.00002$
$0.0^3 3 = 0.0003$
$0.0^2 4 = 0.004$