THE PUBLIC ACCOUNTANTS EXAMINATIONS BOARD

A Committee of the Council of ICPAU

CPA (U) EXAMINATIONS

LEVEL ONE

QUANTITATIVE TECHNIQUES - PAPER 2

TUESDAY 22 AUGUST, 2017

INSTRUCTIONS TO CANDIDATES

1. Time allowed: **3 hours 15 minutes**.
   The first 15 minutes of this examination have been designated for reading time. You may not start to write your answer during this time.
2. This examination contains **six** questions and only **five** questions are to be attempted. Each question carries 20 marks.
3. Formulae and tables are provided on pages 7 – 11.
4. Write your answer to each question on a fresh page in your answer booklet.
5. Please, read further instructions on the answer booklet, before attempting any question.

*Attempt **five** of the **six** questions*

Question 1

(a) Distinguish between simple random sampling and stratified random sampling.

(2 marks)

(b) The following table shows the age range of employees at Abacus Ltd

Age range	20 - 26	26 - 32	32 - 38	38 - 44	44 - 50
No of employees	5	12	x	20	14

Given that the modal age of the employees is 40 years;

Required:

(i) Determine the value of x. **(8 marks)**
(ii) Compute the harmonic mean age. **(6 marks)**
(iii) Represent the data given in the table above on a histogram. **(4 marks)**

(Total 20 marks)

Question 2

(a) Akello, a hotel proprietor in Ntinda Kampala, would like to build either a small hotel or a large hotel. Her plans will depend on the future demand the hotel will serve. She expects either hotel to succeed with a probability of ½. The profits expected from each type of hotel, per annum in million shillings, are shown in the following table.

	Possible future demand	
Type of hotel	Low	High
Large	200	270
Small	160	800

Required:
Using expected monetary value criterion determine the:

(i) best type of hotel Akello should build. **(3 marks)**
(ii) redundant type of hotel Akello should abandon. **(1 mark)**

(b) A sample of 200 CPA students at KAMU College revealed that 18% of them were senior managers and a sample of 400 CPA students at High class College revealed that 15% of them were senior managers.

Required:

Determine whether there is a significant difference between the two proportions in the two colleges at 5% level.

(8 marks)

(c) The canteen at Tina Tana Nursery School sells ice cream packed in three differently shaped containers; cylindrical, conical and cuboid. The following data shows the shapes preferred by 60 children who buy the ice cream.

Shape	No. of children
Cylindrical	17
Conical	24
Cuboid	19

The null hypothesis states that there is no difference between the shapes preferred by the children.

Required:

Test at 5% level whether there is a significant difference between the shapes preferred by the children.

(8 marks)
(Total 20 marks)

Question 3

(a) (i) Identify **two** conditions under which the binomial distribution B(n, p) can be approximated by a normal distribution.

(2 marks)

(ii) A random variable x has the Binomial distribution B(25, 0.38).

Required:

Prove that the distribution can be approximated by a normal distribution.

(2 marks)

(iii) Use the normal approximation to binomial to calculate the probability that x takes on a value more than 15.

(3 marks)

(b) Two fair dice each with faces numbered 1, 2, 3, 4, 5, 6 are tossed together once and the sums of numbers appearing on top are recorded.

Required:

(i) Prepare a possibility space for the sums. **(4 marks)**
(ii) Prepare a probability distribution table for the sums. **(2 marks)**
(iii) Use the probability distribution to compute E(x) for $x \leq 6$. **(2 marks)**

(c) At Nansana brick factory 10% of the bricks produced are found defective. A sample of 15 bricks was chosen at random from the factory.

Required:

Use the Poisson probability distribution to compute the probability that in the sample:

(i) exactly two bricks are defective. **(3 marks)**
(ii) more than one brick are defective. **(2 marks)**

(Total 20 marks)

Question 4

(a) Define the following terms.

(i) Marginal cost. **(1 mark)**
(ii) Index number. **(1 mark)**

(b) A farmer in Lwengo district, who is engaged in agri-business has established that his sales of coffee, y (in kilograms) depends on the number of productive hours, x he dedicates to the business. The expression $y = (2x^3 - 5)^4$ describes the relationship between y and x

Required:

Determine the rate at which the farmer's sales are changing given he dedicates 2 hours.

(4 marks)

(c) Hambe car depot participated in a trade show to popularise a new model of a car in their warehouse. Hambe exhibited q cars and sold all of them. The total cost function (in shillings) experienced by Hambe car depot for the exhibition is given by $C = 2{,}000{,}000 - 1{,}500{,}000q + 25{,}000q^2$.

Required:

Determine the minimum cost of the exhibition. **(5 marks)**

(d) A real estate company based in Kampala collected the following data on average selling prices of three types of houses it sold in the past three years.

Table 1: Average selling price (million shillings)

Year	2 roomed	1 roomed	3 roomed
2014	225	375	440
2015	148	250	390
2016	130	235	400

Table 2: Number of each type of house sold.

Year	2 roomed	1 roomed	3 roomed
2014	42	104	20
2015	28	76	16
2016	32	82	10

Required:

(i) Compute the un-weighted aggregate price index for the houses for 2015 using 2014 as the base year.

(2 marks)

(ii) Compute the Paasche price index for the 2015 and 2016 using 2014 as the base year.

(6 marks)

(iii) Comment on both indices obtained in (d) (i) and (d) (ii) above.

(1 mark)

(Total 20 marks)

Question 5

(a) Identify any **three** limitations of using the graphical method in linear programming problems.

(3 marks)

(b) The staff of Cosmos Ltd proposed an increment in their salary structure based on the length of service in the company. In order to come up with a decision, management computed the duration of service and percentage efficiency scores as shown in the following table.

Length of service (years)	8	12	22	5	15	25	9	10	20	18
Efficiency score (%)	40	20	60	25	42	70	48	51	75	50

Required:

(i) Compute Spearman's rank correlation coefficient. **(5 marks)**

(ii) Comment on their proposal for increment. **(1 mark)**

(c) Dual Company Ltd manufactures two products A and B. The profit contribution per unit of A is Shs 8,000 and that of B is Shs 10,000. There are three machine centres through which the products pass. Product A requires 1 hour machine time in centre 1, 2 hours in centre 2 and 1 hour in centre 3. The machine hour requirements for product B is 1, 1 and 2 respectively in machine centres 1, 2 and 3. The maximum machine time available is 80 hours in machine centre 1, 100 hours in machine centre 2 and 120 hours in machine centre 3. The company wishes to maximize the total contribution from these two products.

Required:

(i) Formulate a linear programming model. **(5 marks)**

(ii) Use the graphical analysis to establish the maximum profit.

(6 marks)

(Total 20 marks)

Question 6

(a) Explain **four** importances of computers in network analysis.

(4 marks)

(b) A project consists of seven activities A, B, C, D, E, D, F and G with the following precedence relationships:

F must follow C, C must follow A, E must follow B and G must follow D. The normal duration, in weeks for each activity, are:

Activity	A	B	C	D	E	F	G
Duration	1	2	2	5	2	2	1

Required:

Construct a Gantt chart for the activities. **(6 marks)**

(c) The table below shows the quantity of sugar produced (in million metric tons) by a sugar factory in 8 years.

Year	2008	2009	2010	2011	2012	2013	2014	2015
Quantity	3	7	6	8	9	7	10	9

Required:

(i) Compute a straight line trend using the least squares method.

(5 marks)

(ii) Use the trend line computed in (c) (i) to estimate the quantity of sugar produced in 2016.

(5 marks)

(Total 20 Marks)

FORMULAE

1. Combination ${}^{n}C_{r} = \frac{n!}{(n-r)!r!}$
2. Permutations ${}^{n}p_{r} = \frac{n!}{(n-r)!}$
3. Mean of the binomial distribution $= np$
4. Variance of the binomial distribution $= np(1-p)$
5. Standard error of population proportion $S_{ps} = \sqrt{\frac{pq}{n}}$
6. Spearman's rank correlation coefficient $r = 1 - \frac{6\sum d^2}{n(n^2-1)}$
7. Product moment coefficient of correlation $= \frac{n\sum xy - \sum x \sum y}{\sqrt{(n\sum x^2 - (\sum x)^2) \times (n\sum y^2 - (\sum y)^2)}}$
8. Cost slope $=$ $\frac{\text{crash cost – normal cost}}{\text{normal time – crash time}}$
9. Harmonic mean (ungrouped data) $hm = \frac{n}{\sum \frac{1}{x}}$
10. Sample mean $\bar{x} = \frac{\sum x}{n}$
11. Harmonic mean (grouped data) $hm = \frac{n}{\sum \frac{f}{x}}$
12. Quartile coefficient of dispersion $= \frac{Q_3 - Q_1}{Q_3 + Q_1}$
13. Mean $\bar{x} = A + \frac{\sum fd}{\sum f}$ or Mean $\bar{x} = \frac{\sum fx}{\sum f}$
14. Median $= Lb + \left(\frac{\frac{N}{2} - Cfb}{fm} \right) C$
15. Mode $= lm + \left(\frac{d_1}{d_1 + d_2} \right) C$
16. Variance $Var(x) = \frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2$

17. Standard deviation $\delta = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2} \quad = \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}}$

18. Sample standard deviation $s = \sqrt{\frac{\sum (x-\bar{x})^2}{n-1}}$

19. Least squares regression equation of y on x is given by; $y = a + bx$

20. Where; $b = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2}$ and $a = \frac{\sum y}{n} - \frac{b\sum x}{n}$

21. Least squares regression equation of x on y is given by; x = c +dy

Where $c = \frac{\sum x}{n} - \frac{d\sum y}{n}$ and $d = \frac{n\sum xy - \sum x\sum y}{n\sum y^2 - (\sum y)^2}$

22. Standardizing normal. $z = \frac{\bar{x} - \mu}{\sigma}$

23. Confidence interval for sample mean $= \bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$

24. $\chi^2 = \sum \frac{(O-E)^2}{E}$

25. Confidence interval of proportion $= p \pm z_{\alpha/2}\sqrt{\frac{pq}{n}}$

26. Pearson coefficient of skewness $Sk = \frac{(\bar{x} - \text{mode})}{s_d}$ or $Sk = \frac{3\left(\bar{x} - median\right)}{s_d}$

27. Expectation = $\sum xP(X = x)$

28. Laspeyres' price index = $\frac{\sum (p_1 \times q_0)}{\sum (q_0 \times p_0)} \times 100$

29. Weighted aggregate price index $= \frac{\sum wv_n}{\sum wv_0} \times 100$

30. Un weighted aggregate price index $= \frac{\sum (p_n)}{\sum (p_0)} \times 100$

31. Additive law of probability; $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

32. Conditional probability $P\left(A/B\right) = \frac{P(A \cap B)}{P(B)}$

33. Independence of A, B $P\left(A/B\right) = P(A) or P(A \cap B) = P(A) \times P(B)$

34. Continuous compounding $A = P(1+r)^n + \frac{b(1+r)^n - b}{r}$

35. Quotient rule of differentiation $f = \frac{vu^1 - uv^1}{v^2}$; where $f = \frac{u}{v}$

36. Paasche's Model $= \frac{\sum(p_1 \times q_1)}{\sum(q_1 \times p_0)} \times 100$

37. Poisson Model $= P(X = x) = e^{-\lambda} \frac{\lambda^x}{x!}$

38. Standard error of the difference of proportion $= \sqrt{\Pi(1-\Pi)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$ where $\Pi = \frac{p_1 n_1 + p_2 n_2}{n_1 + n_2}$

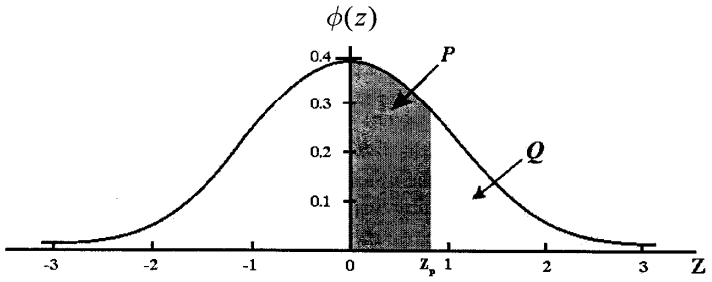
CUMULATIVE NORMAL DISTRIBUTION $P(z)$

Z	0	1	2	3	4	5	6	7	8	9	ADD 1	2	3	4	5	6	7	8	9
0.0	0.0000	0040	0080	0120	0160	0199	0239	0279	0319	0359	4	8	12	16	20	24	28	32	36
0.1	0.0398	0438	0478	0517	0557	0596	0636	0675	0714	0753	4	8	12	16	20	24	28	32	36
0.2	0.0793	0832	0871	0910	0948	0987	1026	1064	1103	1141	4	8	12	15	19	22	27	31	35
0.3	0.1179	1217	1255	1293	1331	1368	1406	1443	1480	1517	4	8	11	15	19	22	26	30	34
0.4	0.1554	1591	1628	1664	1700	1736	1772	1808	1844	1879	4	7	11	14	18	22	25	29	32
0.5	0.1915	1950	1985	2019	2054	2088	2123	2157	2190	2224	3	7	10	14	17	21	24	27	31
0.6	0.2257	2291	2324	2357	2389	2422	2454	2486	2517	2549	3	6	10	13	16	19	23	26	29
0.7	0.2580	2611	2642	2673							3	6	9	12	15	19	22	25	28
					2704	2734	2764	2794	2823	2852	3	6	9	12	15	18	21	24	27
0.8	0.2881	2910	2939	2967	2995	3023					3	6	8	11	14	17	20	22	25
							3051	3078	3106	3133	3	5	8	11	13	16	19	22	24
0.9	0.3159	3186	3212	3238	3264	3289					3	5	8	10	13	16	18	21	23
							3315	3340	3365	3389	2	5	7	10	12	15	17	20	22
1.0	0.3413	3438	3461	3485	3508						2	5	7	10	12	14	17	19	22
						3531	3554	3577	3599	3621	2	4	7	9	11	13	15	18	20
1.1	0.3643	3665	3686	3708							2	4	6	8	11	13	15	17	19
					3729	3749	3770	3790	3810	3830	2	4	6	8	10	12	14	16	18
1.2	0.3849	3869	3888	3907	3925						2	4	6	8	10	11	13	15	17
						3944	3962	3980	3997	4015	2	4	5	7	9	11	13	14	16
1.3	0.4032	4049	4066	4082	4099	4115	4131	4147	4162	4177	2	3	5	6	8	10	11	13	14
1.4	0.4192	4207	4222	4236	4251	4265	4279	4292	4306	4319	1	3	4	6	7	8	10	11	13
1.5	0.4332	4345	4357	4370	4382	4394	4406	4418	4429	4441	1	2	4	5	6	7	8	10	11
1.6	0.4452	4463	4474	4484	4495	4505	4515	4525	4535	4545	1	2	3	4	5	6	7	8	9
1.7	0.4554	4564	4573	4582	4591	4599	4608	4616	4625	4633	1	2	3	3	4	5	6	7	8
1.8	0.4641	4649	4656	4664	4671	4678	4686	4693	4699	4706	1	1	2	3	4	4	5	6	6
1.9	0.4713	4719	4726	4732	4738	4744	4750	4756	4761	4767	1	1	2	2	3	4	4	5	5
2.0	0.4772	4778	4783	4788	4793	4798	4803	4808	4812	4817	0	1	1	2	2	3	3	4	4
2.1	0.4821	4826	4830	4834	4838	4842	4846	4850	4854	4957	0	1	1	2	2	2	3	3	4
2.2	0.4861	4864	4868	4871	4875	4878	4881	4884	4887	4890	0	1	1	1	2	2	2	3	3
2.3	0.4893	4896	4898	4901	4904	4906	4909	4911	4913	4916	0	0	1	1	1	2	2	2	2
2.4	0.4918	4920	4922	4925	4927	4929	4931	4932	4934	4936	0	0	1	1	1	1	1	2	2
2.5	0.4938	4940	4941	4943	4945	4946	4948	4949	4951	4952									
2.6	0.4953	4955	4956	4957	4959	4960	4961	4962	4963	4964									
2.7	0.4965	4966	4967	4968	4969	4970	4971	4972	4973	4974									
2.8	0.4974	4975	4976	4977	4977	4978	4979	4979	4980	4981									
2.9	0.4981	4982	4982	4983	4984	4984	4985	4985	4986	4986									
3.0	0.4987	4990	4993	4995	4997	4998	4998	4999	4999	5000									

The table gives $P(z) = \int_0^z \phi(z)dz$

If the random variable Z is distributed as the standard normal distribution N(0,1) then:

1. $P(o < Z < z_p) = P$(Shaded Area)
2. $P(Z > Z_p) = Q = ½ - P$
3. $P(Z > |Z_p|) = 1 - 2P = 2Q$



PERCENTAGE POINTS OF THE CHI-SQUARE (χ^2) DISTRIBUTION χ^2_Q

	Probability Q									
ν	0.995	0.990	0.975	0.950	0.100	0.050	0.025	0.010	0.005	0.001
1	0.0^4393	0.0^3157	0.0^3982	0.0^2393	2.706	3.841	5.024	6.635	7.879	10.83
2	0.0100	0.0201	0.0506	0.1026	4.605	5.991	7.378	9.210	10.60	13.82
3	0.0717	0.1148	0.2158	0.3518	6.251	7.815	9.348	11.34	12.84	16.27
4	0.2070	0.2971	0.4844	0.7107	7.779	9.488	11.14	13.28	14.86	18.47
5	0.4117	0.5543	0.8312	1.145	9.236	11.07	12.83	15.09	16.75	20.52
6	0.6757	0.8721	1.237	1.635	10.64	12.59	14.45	16.81	18.55	22.46
7	0.9893	1.239	1.690	2.167	12.02	14.07	16.01	18.48	20.28	24.32
8	1.344	1.646	2.180	2.733	13.36	15.51	17.53	20.09	21.95	26.12
9	1.735	2.088	2.700	3.325	14.68	16.92	19.02	21.67	23.59	27.88
10	2.156	2.558	3.247	3.940	15.99	18.31	20.48	23.21	25.19	29.59
11	2.603	3.053	3.816	4.575	17.28	19.68	21.92	24.73	26.76	31.26
12	3.074	3.571	4.404	5.226	18.55	21.03	23.34	26.22	28.30	32.91
13	3.565	4.107	5.009	5.892	19.81	22.36	24.74	27.69	29.82	34.53
14	4.075	4.660	5.629	6.571	21.06	23.68	26.12	29.14	31.32	36.12
15	4.601	5.229	6.262	7.261	22.31	25.00	27.49	30.58	32.80	37.70
16	5.142	5.812	6.908	7.962	23.54	26.30	28.85	32.00	34.27	39.25
17	5.697	6.408	7.564	8.672	24.77	27.59	30.19	33.41	35.72	40.79
18	6.265	7.015	8.231	9.390	25.99	28.87	31.53	34.81	37.16	42.31
19	6.844	7.633	8.907	10.12	27.20	30.14	32.85	36.19	38.58	43.82
20	7.434	8.260	9.591	10.85	28.41	31.41	34.17	37.57	40.00	45.31
21	8.034	8.897	10.28	11.59	29.62	32.67	35.48	38.93	41.40	46.80
22	8.643	9.542	10.98	12.34	30.81	33.92	36.78	40.29	42.80	48.27
23	9.260	10.20	11.69	13.09	32.01	35.17	38.08	41.64	44.18	49.73
24	9.886	10.86	12.40	13.85	33.20	36.42	39.36	42.98	45.56	51.18
25	10.52	11.52	13.12	14.61	34.38	37.65	40.65	44.31	46.93	52.62
26	11.16	12.20	13.84	15.38	35.56	38.89	41.92	45.64	48.29	54.05
27	11.81	12.88	14.57	16.15	36.74	40.11	43.19	46.96	49.64	55.48
28	12.46	13.56	15.31	16.93	37.92	41.34	44.46	48.28	50.99	56.89
29	13.12	14.26	16.05	17.71	39.09	42.56	45.72	49.59	52.34	58.30
30	13.79	14.95	16.79	18.49	40.26	43.77	46.98	50.89	53.67	59.70
40	20.71	22.16	24.43	26.51	51.81	55.76	59.34	63.69	66.77	73.40
50	27.99	29.71	32.36	34.76	63.17	67.50	71.42	76.15	79.49	86.66
60	35.53	37.48	40.48	43.19	74.40	79.08	83.30	88.38	91.95	99.61
70	43.28	45.44	48.76	51.74	85.53	90.53	95.02	100.4	104.2	112.3
80	51.17	53.54	57.15	60.39	96.58	101.9	106.6	112.3	116.3	124.8
90	59.20	61.75	65.65	69.13	107.6	113.1	118.1	124.1	128.3	137.2
100	67.33	70.06	74.22	77.93	118.5	124.3	129.6	135.8	140.2	149.4

The function tabulated is χ^2_Q defined by

$$\int_{\chi^2_Q}^{\infty} f(x)dx = Q; \quad f(x) = \frac{1}{2^{\nu/2}(\nu/2-1)!}x^{\nu/2-1}e^{-x/2}(x>0)$$

where $f(x)$ is the probability density of the χ^2 distribution for ν degrees of freedom. Interpolation ν-wise for $\nu > 30$ gives adequate values (but errors up to 5 units in the last figure may occur for the smaller ν). For $\nu > 100$ the distribution of $\sqrt{(2\chi^2)}$ is approximately normal with mean $\sqrt{(2\nu - 1)}$ and unit variance.

Note: $0.0^4 2 = 0.00002$
$0.0^3 3 = 0.0003$
$0.0^2 4 = 0.004$