

THE PUBLIC ACCOUNTANTS EXAMINATIONS BOARD
A Committee of the Council of ICPAU

CPA (U) EXAMINATIONS

LEVEL ONE

QUANTITATIVE TECHNIQUES – PAPER 2

TUESDAY 16 MARCH, 2021

INSTRUCTIONS TO CANDIDATES:

1. Time allowed: **3 hours 15 minutes**.
The first 15 minutes of this examination have been designated for reading time. You may not start to write your answer during this time.
2. This examination contains **six** question and only **five** questions are to be attempted. Each question carries 20 marks.
3. Formulae and tables are provided on pages 9 – 14.
4. Write your answer to each question on a fresh page in your answer booklet.
5. Please, read further instructions on the answer booklet, before attempting any question.

Attempt five of the six questions

Question 1

- (a) (i) Distinguish between discrete and continuous data. **(2 marks)**
 (ii) Explain the advantages of grouping data. **(2 marks)**
 (b) The daily hours of sunshine in Kampala from 1st August 2020 to 9th September 2020 from the metrological department in Entebbe were as shown below:

| | | | | | | | | | |
|-----|-----|------|------|------|-----|-----|------|------|-----|
| 7.0 | 7.6 | 12.5 | 12.9 | 8.3 | 9.7 | 8.3 | 11.1 | 7.5 | 8.1 |
| 7.5 | 9.8 | 10.4 | 11.6 | 11.3 | 7.3 | 7.8 | 6.5 | 6.2 | 8.4 |
| 6.1 | 5.6 | 5.6 | 5.8 | 4.8 | 4.3 | 3.0 | 2.6 | 2.8 | 3.9 |
| 2.6 | 3.2 | 2.4 | 2.6 | 9.2 | 7.3 | 7.0 | 6.5 | 10.1 | 5.5 |

A district Agricultural officer was interested in advising the farmers about the weather conditions that favour drying of their produce.

Required:

- (i) Using uniform class intervals and starting with the class 2.0 - 2.9; construct a frequency distribution table for the data above.

(3 marks)

From the table above, calculate the following:

- (ii) Average number of hours of sunshine per day. **(2 marks)**
 (iii) Co-efficient of variation and comment on the result. **(6 marks)**
 (c) ALD a non-governmental organization has carried out a research on the age distribution for teachers in school A and school B in the newly created district of Mbikwe in Eastern Uganda. The results are as shown in the table below.

| Age | Frequencies | |
|------|-------------|----------|
| | School A | School B |
| 20 - | 4 | 0 |
| 25 - | 6 | 2 |
| 30 - | 11 | 4 |
| 35 - | 14 | 7 |
| 40 - | 9 | 11 |
| 45 - | 5 | 12 |
| 50 - | 5 | 11 |
| 55 - | 3 | 8 |
| 60 - | 0 | 5 |

Required:

- (i) Draw frequency polygons on the same graph to compare the age distributions of the teachers in school A and B.

(4 marks)

- (ii) Comment on your results.

(1 mark)**(Total 20 marks)****Question 2**

- (a) Explain the characteristics of a binomial distribution. **(2 marks)**

- (b) CICL LTD has recently opened a chocolate company in Namanve industrial park. Due to inexperience of the newly recruited staff, 10% of the chocolates produced in the factory are mis-shaped. In a sample of 1000 chocolates, find the probability that the number of mis-shapes is:

- (i) Less than 80. **(2 marks)**

- (ii) Between 90 and 115 inclusive. **(3 marks)**

- (c) The manager of VHL has established that the number of customers visiting his office follows a discrete random variable r with probability $P(r)$ defined by

$$P(r) = kr^2 \quad r = 1, 2, 3$$

$$P(r) = k(7-r)^2, \quad r = 4, 5, 6$$

$$P(r) = 0, \text{ otherwise}$$

Required:

Determine;

- (i) The value of k **(3 marks)**

- (ii) The mean number of customers that visit the manager's office.

(4 marks)

- (d) NELN General Services Ltd a dealer in whole sale business receives customers at an average rate of 4 customers per minute. Assuming that customers' arrivals follow a Poisson distribution;

Required:

Calculate the probability that:

- (i) No customer arrives in any particular minute. **(2 marks)**

- (ii) Two or more customers arrive in any particular minute. **(4 marks)**

(Total 20 marks)

Question 3

- (a) Explain the following terms as applied in hypothesis testing.
- (i) Hypothesis (1 mark)
 - (ii) Critical value (1 mark)
 - (iii) Null hypothesis (1 mark)
- (b) In a certain meeting, the guests were served with the breakfast of cereals with constituents A, B, C, and D. The cost of the cereals in Uganda shillings per kilogram in July, August 2020 and quantities needed were as shown in the table below.

| | A | B | C | D |
|----------------|-----|-------|-------|-------|
| July | | | | |
| Prices (Shs) | 800 | 1,050 | 1,200 | 800 |
| Quantities(kg) | 6 | 4 | 3 | 2 |
| August | | | | |
| Prices (Shs) | 880 | 1,260 | 1,500 | 1,080 |
| Quantities(kg) | 8 | 5 | 2 | 3 |

Required:

Using July as the base month:

- (i) Calculate the simple aggregate price index for August 2020. (2 marks)
 - (ii) Compute Laspreyres quantity index for August 2020 and comment on the result. (4 marks)
- (c) Kalilo Farmers Association deals in the growing and production of maize, they need to insure their produce against the unexpected weather patterns. The last released information from national forecasters show that the probability of receiving rain in the next six months is 0.75. They can decide to insure their produce in order to attract investors or fail to insure such that they sell on the local market. Using past records, their sales during a rainy season fetch 10 million if insured otherwise 8 million. If there is no rain, the sales fetch 9 million if insured otherwise only 6 million as shown below:

| Decision alternatives | states of nature | |
|-----------------------|------------------|-----------|
| | Rain | No Rain |
| Insure | 10,000,000 | 9,000,000 |
| Not Insure | 8,000,000 | 6,000,000 |

Required:

- Demonstrate if the farmers should insure their produce? **(5 marks)**
- (c) The personnel manager of a large firm is investigating whether there is any association between the length of service of the employees and the type of training they receive. A random sample of 200 employee records is taken from the last few years and is classified according to these criteria. Length of service is classified as short; (less than one year), medium (1- 3 years) and long (more than 3 years) type of training is classified as being merely an initial induction, on job and continuous training. The data is as follows.

| | | Length of service | | |
|------------------|---------------------|-------------------|--------|------|
| | | short | medium | long |
| Type of training | Induction Course | 26 | 30 | 24 |
| | On job & continuous | 28 | 32 | 60 |

Required:

Using Chi - square, examine at the 5% level of significance whether this data provide evidence of association between length of service and type of training, stating clearly your null alternative hypothesis.

(6 marks)**(Total 20 marks)****Question 4**

- (a) Explain the applications of linear regression. **(3 marks)**
- (b) The Manager of a certain office supervises 10 clerical assistants, each using the word-processor. The assistants did not receive the same training in the use of word-processor. In order to make an assessment of the need for training, the manager monitored their work during a given week recording the number of pieces of work correctly produced (x) and the number of days spent on training (y).

The results are summarised in the table below.

| | | | | | | | | | | |
|-----|----|----|----|----|----|----|----|----|----|----|
| x | 35 | 26 | 33 | 22 | 40 | 31 | 22 | 20 | 24 | 23 |
| y | 10 | 2 | 7 | 5 | 11 | 8 | 9 | 3 | 8 | 2 |

Required:

- (i) Determine the equation x on y by least squares method. **(8 marks)**
- (ii) Using the equation, find the number of pieces of work produced correctly if a person is trained for 14 days. **(1 mark)**

- (c) ABC Ltd is the leading distributor of Yamaha motor cycles in Uganda and their sales as at November 2020 are as shown in the table below.

| Months | Sales |
|-----------|-------|
| Jan | 540 |
| Feb | 550 |
| March | 660 |
| April | 510 |
| May | 480 |
| June | 500 |
| July | 470 |
| August | 460 |
| September | 510 |
| October | 550 |
| November | 590 |

In 2020, the company recruited a new General Manager and he would wish to make forecasts based on the previous sales.

Required:

Using smoothing constant $a = 0.2$ and January sales as February forecasts,

- (i) Find the forecasted sales corresponding to the year 2021.

(7 marks)

- (ii) Using (c) (i), find the forecast sales for January 2022.

(1 mark)

(Total 20 marks)

Question 5

- (a) XYZ is the leading distributor of Tampako drink. Their fixed costs are Shs 660 per unit. The variable costs are given by the function $V(x) = 250 - 2x^2$, where x is the number of units produced per day.

Required:

- (i) Derive a function to determine the total costs for a day. **(2 marks)**

- (ii) Find the minimum costs per day. **(4 marks)**

- (b) ABIB Ltd specialises in the production of hedges (x) and doors (y). The raw material absorption of the two products per day follows a production function given by $y = 4x - 2x^2$.

Required:

- (i) Sketch a graph of $y = 4x - 2x^2$ (8 marks)
- (ii) From the graph, find the maximum number of hedges and doors that can be produced in the above factory per day. (1 mark)
- (c) A new car dealer in Namanve industrial park assembles two types of cars, Sunny and Hiace. On a certain day he assembled 4 Sunny and 2 Hiace at a total cost of Shs 80,000,000. The following day he assembled 3 Sunny cars one Hiace at a total cost of Shs 50,000,000.

Required:

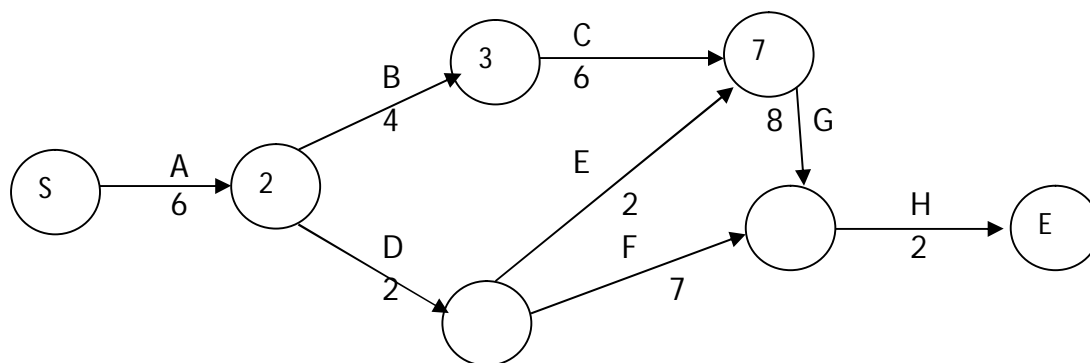
Using matrix method, find the total cost of assembling each car.

(5 marks)

(Total 20 marks)

Question 6

- (a) Explain the following terms as applied in linear programming.
- (i) Dual (1 mark)
- (ii) Primal (1 mark)
- (b) COMBO Ltd is a transport company specializing in the delivery of cement in the East African region. It has been doing all its operations using the manual system. During the recently concluded audit, the auditors advised the management to embrace application of technology in their operations. The managing Director hired a consultant from South Africa to develop for him a software that can help him manage his fleet well. The consultant has sent him a schedule of activities on a network as shown below.



The Managing Director has approached you for interpretation of the above network.

Required:

- (i) Present the above information in a tabular form clearly indicating the activities, preceding activities and the activity duration in days. **(4 marks)**
- (ii) Determine the critical path. **(2 marks)**
- (c) Jumary Company Ltd recycles two types of polythene bags x_1 and x_2 in varying proportions to produce three types of plastic cups A , B and C . He wishes to produce at least 150 units of A , 200 units of B and 60 units of C . Each ton of x_1 yields 3 units of A , 5 of B and 3 of C . Each ton of x_2 yields 5 units of A , 5 of B and 1 of C . if x_1 costs Shs 40,000 per ton and x_2 Shs 50,000 per ton,

Required:

- (i) Formulate a linear programming model for the above data. **(3 marks)**
 - (ii) Generate the dual for the linear programming model in (b) (i) above. **(3marks)**
 - (iii) Solve the dual in (b) (ii) above to determine the minimum costs. **(6 marks)**
- (Total 20 marks)**

FORMULAE

1. Combination ${}^nC_r = \frac{n!}{(n-r)!r!}$
2. Permutations ${}^nP_r = \frac{n!}{(n-r)!}$
3. Mean of the binomial distribution = np
4. Standard deviation = \sqrt{npq}
5. Variance of the binomial distribution = $np(1-p)$
6. Standard error of population proportion $S_{ps} = \sqrt{\frac{pq}{n}}$
7. Spearman's rank correlation coefficient $r = 1 - \frac{6\sum d^2}{n(n^2-1)}$
8. Product moment coefficient of correlation =
$$\frac{n\sum xy - \sum x \sum y}{\sqrt{(n\sum x^2 - (\sum x)^2) \times (n\sum y^2 - (\sum y)^2)}}$$
9. Cost slope =
$$\frac{\text{crash cost} - \text{normal cost}}{\text{normal time} - \text{crash time}}$$
10. Harmonic mean (ungrouped data) $hm = \frac{n}{\sum \frac{1}{x}}$
11. Sample mean $\bar{x} = \frac{\sum x}{n}$
12. Harmonic mean (grouped data) $hm = \frac{n}{\sum \frac{f}{x}}$
13. Quartile coefficient of dispersion = $\frac{Q_3 - Q_1}{Q_3 + Q_1}$
14. Bowley's coefficient of skewness = $\frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$
15. Mean $\bar{x} = A + \frac{\sum fd}{\sum f}$ or Mean $\bar{x} = \frac{\sum fx}{\sum f}$

16. Median $= Lb + \left(\frac{\frac{N}{2} - Cfb}{fm} \right) C$
17. Mode $= lm + \left(\frac{d_1}{d_1 + d_2} \right) C$
18. Variance $Var(x) = \frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2$
19. Standard deviation $\delta = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2} = \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}}$
20. Sample standard deviation $s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$
21. Least squares regression equation of y on x is given by; $y = a + bx$
Where; $b = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2}$ and $a = \frac{\sum y}{n} - b \frac{\sum x}{n}$
22. Least squares regression equation of x on y is given by; $x = c + dy$
Where $c = \frac{\sum x}{n} - d \frac{\sum y}{n}$ and $d = \frac{n \sum xy - \sum x \sum y}{n \sum y^2 - (\sum y)^2}$
23. Standardising normal. $z = \frac{\bar{x} - \mu}{\sigma}$
24. Confidence interval for sample mean $= \bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$
25. $\chi^2 = \sum \frac{(O - E)^2}{E}$
26. Confidence interval of proportion $= p \pm z_{\alpha/2} \sqrt{\frac{pq}{n}}$
27. Pearson coefficient of skewness $Sk = \frac{(\bar{x} - \text{mode})}{s_d}$ or $Sk = \frac{3(\bar{x} - \text{median})}{s_d}$
28. Expectation $= \sum xP(X = x)$
29. Laspeyres' price index $= \frac{\sum (p_1 \times q_0)}{\sum (q_0 \times p_0)} \times 100$

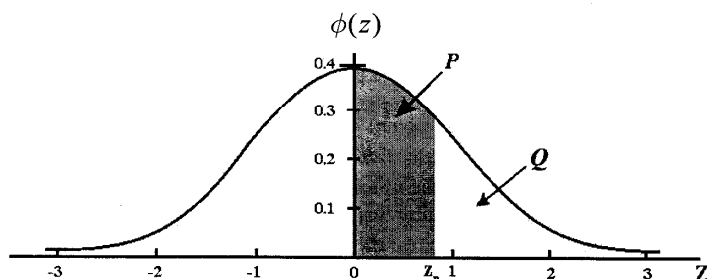
30. Paasche's Model $= \frac{\sum (p_1 \times q_1)}{\sum (q_1 \times p_0)} \times 100$
31. Weighted aggregate price index $= \frac{\sum wv_n}{\sum wv_0} \times 100$
32. Additive law of probability; $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
33. Conditional probability $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$
34. Independence of A, B $P\left(\frac{A}{B}\right) = P(A)$ or $P(A \cap B) = P(A) \times P(B)$
35. Continuous compounding $A = P(1+r)^n + \frac{b(1+r)^n - b}{r}$
36. Quotient rule of differentiation $f = \frac{vu^1 - uv^1}{v^2}$; where $f = \frac{u}{v}$
37. Poisson Model $P(X = x) = e^{-\lambda} \frac{\lambda^x}{x!}$

| CUMULATIVE NORMAL DISTRIBUTION $P(z)$ | | | | | | | | | | | ADD | | | | | | | | |
|---------------------------------------|--------|------|------|------|------|------|------|------|------|------|-----|---|----|----|----|----|----|----|----|
| Z | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 0.0 | 0.0000 | 0040 | 0080 | 0120 | 0160 | 0199 | 0239 | 0279 | 0319 | 0359 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 0.1 | 0.0398 | 0438 | 0478 | 0517 | 0557 | 0596 | 0636 | 0675 | 0714 | 0753 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| 0.2 | 0.0793 | 0832 | 0871 | 0910 | 0948 | 0987 | 1026 | 1064 | 1103 | 1141 | 4 | 8 | 12 | 15 | 19 | 22 | 27 | 31 | 35 |
| 0.3 | 0.1179 | 1217 | 1255 | 1293 | 1331 | 1368 | 1406 | 1443 | 1480 | 1517 | 4 | 8 | 11 | 15 | 19 | 22 | 26 | 30 | 34 |
| 0.4 | 0.1554 | 1591 | 1628 | 1664 | 1700 | 1736 | 1772 | 1808 | 1844 | 1879 | 4 | 7 | 11 | 14 | 18 | 22 | 25 | 29 | 32 |
| 0.5 | 0.1915 | 1950 | 1985 | 2019 | 2054 | 2088 | 2123 | 2157 | 2190 | 2224 | 3 | 7 | 10 | 14 | 17 | 21 | 24 | 27 | 31 |
| 0.6 | 0.2257 | 2291 | 2324 | 2357 | 2389 | 2422 | 2454 | 2486 | 2517 | 2549 | 3 | 6 | 10 | 13 | 16 | 19 | 23 | 26 | 29 |
| 0.7 | 0.2580 | 2611 | 2642 | 2673 | 2704 | 2734 | 2764 | 2794 | 2823 | 2852 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| 0.8 | 0.2881 | 2910 | 2939 | 2967 | 2995 | 3023 | 3051 | 3078 | 3106 | 3133 | 3 | 6 | 8 | 11 | 14 | 17 | 20 | 22 | 25 |
| 0.9 | 0.3159 | 3186 | 3212 | 3238 | 3264 | 3289 | 3315 | 3340 | 3365 | 3389 | 3 | 5 | 8 | 11 | 13 | 16 | 19 | 22 | 24 |
| 1.0 | 0.3413 | 3438 | 3461 | 3485 | 3508 | 3531 | 3554 | 3577 | 3599 | 3621 | 3 | 5 | 7 | 10 | 12 | 15 | 17 | 20 | 22 |
| 1.1 | 0.3643 | 3665 | 3686 | 3708 | 3729 | 3749 | 3770 | 3790 | 3810 | 3830 | 2 | 4 | 7 | 9 | 11 | 13 | 15 | 18 | 20 |
| 1.2 | 0.3849 | 3869 | 3888 | 3907 | 3925 | 3944 | 3962 | 3980 | 3997 | 4015 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| 1.3 | 0.4032 | 4049 | 4066 | 4082 | 4099 | 4115 | 4131 | 4147 | 4162 | 4177 | 2 | 4 | 6 | 8 | 10 | 11 | 13 | 15 | 17 |
| 1.4 | 0.4192 | 4207 | 4222 | 4236 | 4251 | 4265 | 4279 | 4292 | 4306 | 4319 | 2 | 3 | 5 | 6 | 8 | 10 | 11 | 13 | 14 |
| 1.5 | 0.4332 | 4345 | 4357 | 4370 | 4382 | 4394 | 4406 | 4418 | 4429 | 4441 | 1 | 3 | 4 | 5 | 6 | 7 | 8 | 10 | 11 |
| 1.6 | 0.4452 | 4463 | 4474 | 4484 | 4495 | 4505 | 4515 | 4525 | 4535 | 4545 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1.7 | 0.4554 | 4564 | 4573 | 4582 | 4591 | 4599 | 4608 | 4616 | 4625 | 4633 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 |
| 1.8 | 0.4641 | 4649 | 4656 | 4664 | 4671 | 4678 | 4686 | 4693 | 4699 | 4706 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 6 |
| 1.9 | 0.4713 | 4719 | 4726 | 4732 | 4738 | 4744 | 4750 | 4756 | 4761 | 4767 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 2.0 | 0.4772 | 4778 | 4783 | 4788 | 4793 | 4798 | 4803 | 4808 | 4812 | 4817 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 2.1 | 0.4821 | 4826 | 4830 | 4834 | 4838 | 4842 | 4846 | 4850 | 4854 | 4857 | 0 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 4 |
| 2.2 | 0.4861 | 4864 | 4868 | 4871 | 4875 | 4878 | 4881 | 4884 | 4887 | 4890 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 |
| 2.3 | 0.4893 | 4896 | 4898 | 4901 | 4904 | 4906 | 4909 | 4911 | 4913 | 4916 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| 2.4 | 0.4918 | 4920 | 4922 | 4925 | 4927 | 4929 | 4931 | 4932 | 4934 | 4936 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 2 | 2 |
| 2.5 | 0.4938 | 4940 | 4941 | 4943 | 4945 | 4946 | 4948 | 4949 | 4951 | 4952 | | | | | | | | | |
| 2.6 | 0.4953 | 4955 | 4956 | 4957 | 4959 | 4960 | 4961 | 4962 | 4963 | 4964 | | | | | | | | | |
| 2.7 | 0.4965 | 4966 | 4967 | 4968 | 4969 | 4970 | 4971 | 4972 | 4973 | 4974 | | | | | | | | | |
| 2.8 | 0.4974 | 4975 | 4976 | 4977 | 4977 | 4978 | 4979 | 4979 | 4980 | 4981 | | | | | | | | | |
| 2.9 | 0.4981 | 4982 | 4982 | 4983 | 4984 | 4984 | 4985 | 4985 | 4986 | 4986 | | | | | | | | | |
| 3.0 | 0.4987 | 4990 | 4993 | 4995 | 4997 | 4998 | 4998 | 4999 | 4999 | 5000 | | | | | | | | | |

The table gives $P(z) = \int_0^z \phi(z) dz$

If the random variable Z is distributed as the standard normal distribution $N(0,1)$ then:

1. $P(0 < Z < z_p) = P(\text{Shaded Area})$
2. $P(Z > Z_p) = Q = \frac{1}{2} - P$
3. $P(Z > |Z_p|) = 1 - 2P = 2Q$



PERCENTAGE POINTS OF THE CHI-SQUARE (χ^2) DISTRIBUTION χ^2_Q

| Probability Q | | | | | | | | | | |
|---------------|----------|----------|----------|----------|-------|-------|-------|-------|-------|-------|
| ν | 0.995 | 0.990 | 0.975 | 0.950 | 0.100 | 0.050 | 0.025 | 0.010 | 0.005 | 0.001 |
| 1 | 0.004393 | 0.004575 | 0.004982 | 0.005393 | 2.706 | 3.841 | 5.024 | 6.635 | 7.879 | 10.83 |
| 2 | 0.0100 | 0.0201 | 0.0506 | 0.1026 | 4.605 | 5.991 | 7.378 | 9.210 | 10.60 | 13.82 |
| 3 | 0.0717 | 0.1148 | 0.2158 | 0.3518 | 6.251 | 7.815 | 9.348 | 11.34 | 12.84 | 16.27 |
| 4 | 0.2070 | 0.2971 | 0.4844 | 0.7107 | 7.779 | 9.488 | 11.14 | 13.28 | 14.86 | 18.47 |
| 5 | 0.4117 | 0.5543 | 0.8312 | 1.145 | 9.236 | 11.07 | 12.83 | 15.09 | 16.75 | 20.52 |
| 6 | 0.6757 | 0.8721 | 1.237 | 1.635 | 10.64 | 12.59 | 14.45 | 16.81 | 18.55 | 22.46 |
| 7 | 0.9893 | 1.239 | 1.690 | 2.167 | 12.02 | 14.07 | 16.01 | 18.48 | 20.28 | 24.32 |
| 8 | 1.344 | 1.646 | 2.180 | 2.733 | 13.36 | 15.51 | 17.53 | 20.09 | 21.95 | 26.12 |
| 9 | 1.735 | 2.088 | 2.700 | 3.325 | 14.68 | 16.92 | 19.02 | 21.67 | 23.59 | 27.88 |
| 10 | 2.156 | 2.558 | 3.247 | 3.940 | 15.99 | 18.31 | 20.48 | 23.21 | 25.19 | 29.59 |
| 11 | 2.603 | 3.053 | 3.816 | 4.575 | 17.28 | 19.68 | 21.92 | 24.73 | 26.76 | 31.26 |
| 12 | 3.074 | 3.571 | 4.404 | 5.226 | 18.55 | 21.03 | 23.34 | 26.22 | 28.30 | 32.91 |
| 13 | 3.565 | 4.107 | 5.009 | 5.892 | 19.81 | 22.36 | 24.74 | 27.69 | 29.82 | 34.53 |
| 14 | 4.075 | 4.660 | 5.629 | 6.571 | 21.06 | 23.68 | 26.12 | 29.14 | 31.32 | 36.12 |
| 15 | 4.601 | 5.229 | 6.262 | 7.261 | 22.31 | 25.00 | 27.49 | 30.58 | 32.80 | 37.70 |
| 16 | 5.142 | 5.812 | 6.908 | 7.962 | 23.54 | 26.30 | 28.85 | 32.00 | 34.27 | 39.25 |
| 17 | 5.697 | 6.408 | 7.564 | 8.672 | 24.77 | 27.59 | 30.19 | 33.41 | 35.72 | 40.79 |
| 18 | 6.265 | 7.015 | 8.231 | 9.390 | 25.99 | 28.87 | 31.53 | 34.81 | 37.16 | 42.31 |
| 19 | 6.844 | 7.633 | 8.907 | 10.12 | 27.20 | 30.14 | 32.85 | 36.19 | 38.58 | 43.82 |
| 20 | 7.434 | 8.260 | 9.591 | 10.85 | 28.41 | 31.41 | 34.17 | 37.57 | 40.00 | 45.31 |
| 21 | 8.034 | 8.897 | 10.28 | 11.59 | 29.62 | 32.67 | 35.48 | 38.93 | 41.40 | 46.80 |
| 22 | 8.643 | 9.542 | 10.98 | 12.34 | 30.81 | 33.92 | 36.78 | 40.29 | 42.80 | 48.27 |
| 23 | 9.260 | 10.20 | 11.69 | 13.09 | 32.01 | 35.17 | 38.08 | 41.64 | 44.18 | 49.73 |
| 24 | 9.886 | 10.86 | 12.40 | 13.85 | 33.20 | 36.42 | 39.36 | 42.98 | 45.56 | 51.18 |
| 25 | 10.52 | 11.52 | 13.12 | 14.61 | 34.38 | 37.65 | 40.65 | 44.31 | 46.93 | 52.62 |
| 26 | 11.16 | 12.20 | 13.84 | 15.38 | 35.56 | 38.89 | 41.92 | 45.64 | 48.29 | 54.05 |
| 27 | 11.81 | 12.88 | 14.57 | 16.15 | 36.74 | 40.11 | 43.19 | 46.96 | 49.64 | 55.48 |
| 28 | 12.46 | 13.56 | 15.31 | 16.93 | 37.92 | 41.34 | 44.46 | 48.28 | 50.99 | 56.89 |
| 29 | 13.12 | 14.26 | 16.05 | 17.71 | 39.09 | 42.56 | 45.72 | 49.59 | 52.34 | 58.30 |
| 30 | 13.79 | 14.95 | 16.79 | 18.49 | 40.26 | 43.77 | 46.98 | 50.89 | 53.67 | 59.70 |
| 40 | 20.71 | 22.16 | 24.43 | 26.51 | 51.81 | 55.76 | 59.34 | 63.69 | 66.77 | 73.40 |
| 50 | 27.99 | 29.71 | 32.36 | 34.76 | 63.17 | 67.50 | 71.42 | 76.15 | 79.49 | 86.66 |
| 60 | 35.53 | 37.48 | 40.48 | 43.19 | 74.40 | 79.08 | 83.30 | 88.38 | 91.95 | 99.61 |
| 70 | 43.28 | 45.44 | 48.76 | 51.74 | 85.53 | 90.53 | 95.02 | 100.4 | 104.2 | 112.3 |
| 80 | 51.17 | 53.54 | 57.15 | 60.39 | 96.58 | 101.9 | 106.6 | 112.3 | 116.3 | 124.8 |
| 90 | 59.20 | 61.75 | 65.65 | 69.13 | 107.6 | 113.1 | 118.1 | 124.1 | 128.3 | 137.2 |
| 100 | 67.33 | 70.06 | 74.22 | 77.93 | 118.5 | 124.3 | 129.6 | 135.8 | 140.2 | 149.4 |

The function tabulated is χ^2_Q defined by

$$\int_{\chi^2_Q}^{\infty} f(x) dx = Q; \quad f(x) = \frac{1}{2^{1/2} (\frac{1}{2} \nu - 1)!} x^{1/2 - 1} e^{-x/2} (x > 0)$$

where $f(x)$ is the probability density of the χ^2 distribution for ν degrees of freedom.

Interpolation ν -wise for $\nu > 30$ gives adequate values (but errors up to 5 units in the last figure may occur for the smaller ν). For $\nu > 100$ the distribution of $\sqrt{(2 \chi^2)}$ is approximately normal with mean $\sqrt{(2\nu - 1)}$ and unit variance.

Note: $0.0^4 = 0.00002$
 $0.0^3 = 0.0003$
 $0.0^2 = 0.004$

PERCENTAGE POINTS OF STUDENT'S t -DISTRIBUTION t_Q

| ν | Probability* | | | | | | | | | Q $2Q$ |
|----------|--------------|-------|-------|-------|-------|-------|--------|-------|--------|-------------|
| | 0.25 | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 | 0.0025 | 0.001 | 0.0005 | |
| | 0.50 | 0.20 | 0.10 | 0.050 | 0.02 | 0.010 | 0.0050 | 0.002 | 0.0010 | |
| 1 | 1.000 | 3.078 | 6.314 | 12.71 | 31.82 | 63.66 | 127.3 | 318.3 | 636.6 | |
| 2 | 0.816 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 14.09 | 22.33 | 31.60 | |
| 3 | 0.765 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 7.453 | 10.21 | 12.92 | |
| 4 | 0.741 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 5.598 | 7.173 | 8.610 | |
| 5 | 0.727 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 4.773 | 5.893 | 6.869 | |
| 6 | 0.718 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 4.317 | 5.208 | 5.959 | |
| 7 | 0.711 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 4.029 | 4.785 | 5.408 | |
| 8 | 0.706 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 3.833 | 4.501 | 5.041 | |
| 9 | 0.703 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 3.690 | 4.297 | 4.781 | |
| 10 | 0.700 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 3.581 | 4.144 | 4.587 | |
| 11 | 0.697 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 3.497 | 4.025 | 4.437 | |
| 12 | 0.695 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 3.428 | 3.930 | 4.318 | |
| 13 | 0.694 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 3.372 | 3.852 | 4.221 | |
| 14 | 0.692 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 3.326 | 3.787 | 4.140 | |
| 15 | 0.691 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 3.286 | 3.733 | 4.073 | |
| 16 | 0.690 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 3.252 | 3.686 | 4.015 | |
| 17 | 0.689 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 3.222 | 3.646 | 3.965 | |
| 18 | 0.688 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 3.197 | 3.610 | 3.922 | |
| 19 | 0.688 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 3.174 | 3.579 | 3.883 | |
| 20 | 0.687 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 3.153 | 3.552 | 3.850 | |
| 21 | 0.686 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.135 | 3.527 | 3.819 | |
| 22 | 0.686 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 3.119 | 3.505 | 3.792 | |
| 23 | 0.685 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 3.104 | 3.485 | 3.767 | |
| 24 | 0.685 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 3.091 | 3.467 | 3.745 | |
| 25 | 0.684 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 3.078 | 3.450 | 3.725 | |
| 26 | 0.684 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 3.067 | 3.435 | 3.707 | |
| 27 | 0.684 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 3.057 | 3.421 | 3.690 | |
| 28 | 0.683 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 3.047 | 3.408 | 3.674 | |
| 29 | 0.683 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 3.038 | 3.396 | 3.659 | |
| 30 | 0.683 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 3.030 | 3.385 | 3.646 | |
| 40 | 0.681 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 2.971 | 3.307 | 3.551 | |
| 60 | 0.679 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 | 2.915 | 3.232 | 3.460 | |
| 120 | 0.677 | 1.289 | 1.658 | 1.980 | 2.358 | 2.617 | 2.860 | 3.160 | 3.373 | |
| ∞ | 0.674 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 2.807 | 3.090 | 3.291 | |

The function tabulated is t_Q defined by

$$\int_{t_Q}^{\infty} f(t) dt = Q; \quad f(t) = \frac{(\frac{1}{2}\nu - \frac{1}{2})!}{\sqrt{(v\pi)(\frac{1}{2}\nu - 1)!}} \cdot \frac{1}{(1 + t^2/\nu)^{(\nu+1)/2}}$$

where $f(t)$ is the probability density of the t -distribution.

Interpolation ν -wise should be linear in $120/\nu$ for $\nu > 30$.

Use (i) upper row for one tail-tests

(i) lower row for two tail-tests

If x is a random variable with the t -probability distribution for ν degrees of freedom, the probability that $x > t_Q$ is Q and the probability that $|x| > t_Q$ is $2Q$.

The graph shows the form of the distribution for $\nu = 2$. The shaded area represents the probability Q . For large ν the distribution approximates to the normal distribution $N(0,1)$, shown by the dotted line.

