

# THE PUBLIC ACCOUNTANTS EXAMINATIONS BOARD

*A Committee of the Council of ICPAU*

## CPA(U) EXAMINATIONS

### LEVEL ONE

#### QUANTITATIVE TECHNIQUES - PAPER 2

**TUESDAY 20 AUGUST, 2019**

#### INSTRUCTIONS TO CANDIDATES

1. Time allowed: **3 hours 15 minutes**.  
The first 15 minutes of this examination have been designated for reading time. You may not start to write your answer during this time.
2. This examination contains **six** questions and only **five** questions are to be attempted. Each question carries 20 marks.
3. Formulae and tables are provided on pages 8 – 13.
4. Write your answer to each question on a fresh page in your answer booklet.
5. Please, read further instructions on the answer booklet, before attempting any question.

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*Attempt five of the six questions*

### Question 1

- (a) Define and give the advantages of quasi random sampling technique. **(2 marks)**
- (b) The following table shows the distribution of monthly wages of 100 workers at a manufacturing plant.

Wage (Shs '000')	No of workers
300 - 499	6
500 - 699	24
700 - 899	30
900 - 1,099	28
1,100 - 1,299	9
1,300 - 1,499	3

#### Required:

- (i) Construct the cumulative frequency curve and use it to estimate the median, lower quartile and the upper quartile wage **(11 marks)**
- (ii) Compute quartile coefficient of dispersion. **(2 marks)**
- (iii) Compute the mean wage. **(5 marks)**
- (Total 20 marks)**

### Question 2

- (a) According to the Human Resource Manual of TID Company, all employees are required to enroll on any of the available medical schemes (A or B). Statistics show that 65% of the employees are enrolled to scheme A and 40% of these are male. If only 55% of those enrolled on scheme B are male;

#### Required:

Determine the probability that an employee chosen at random is:

- (ii) a male employee. **(3 marks)**
- (iii) enrolled to scheme A given that he is a male employee. **(3 marks)**
- (b) From a normal population of razor blade packets with mean mass,  $\mu$  and variance 4.5g, a random sample of 10 packets was taken and the mass in, g, was found to be: 12.9, 12.3, 16.4, 12.6, 13.1, 13.7, 12.5, 10.8, 15.5, and 11.2.

**Required:**

Compute the 95% confidence limits for the mean  $\mu$  and comment on the results.

**(6 marks)**

- (c) A survey was carried out to establish the impact of the dependence on loan financing and equity financing on companies' growth. The results from a total of 180 small, medium and large companies revealed the following.

	Company size		
	Small	Medium	Large
Loan financing	21	36	30
Equity financing	48	26	19

**Required:**

Using Chi square, test the hypothesis at 5% level of significance that the growth of a company is independent of its type of financing.

**(8 marks)****(Total 20 marks)****Question 3**

- (a) The following table shows items that constitute the food basket for Azimio community. The prices and quantities for the base and current year are as shown in the following table.

Item	Base year		Current year	
	Price (Shs '000')	Quantity(kg)	Price (Shs '000')	Quantity(kg)
A	10	25	15	30
B	15	30	10	20
C	20	10	25	20
D	25	25	50	50
E	30	50	35	35
F	10	25	30	20

**Required:**

Calculate Laspeyre's index for the above information and comment on your result.

**(6 marks)**

- (b) Tweyambe is a private enterprise that deals in the manufacture of super concrete products demanded by construction companies. Past records show that the quarterly demand, in metric tons, for the last 3 years is as shown in the following table:

Year	Quarter	Demand
2016	1	390
	2	425
	3	420
	4	475
2017	1	440
	2	460
	3	465
	4	500
2018	1	455
	2	520
	3	495
	4	560

**Required:**

- (i) Distinguish between positive correlation and negative correlation giving an example for each category. **(4 marks)**
- (ii) Compute the correlation coefficient using product moment coefficient method. **(9 marks)**
- (iii) Comment on the value of the correlation coefficient obtained in (b) (ii) above. **(1 mark)**

**(Total 20 marks)****Question 4**

- (a) Distinguish between the terms 'fixed costs' and 'variable costs'. **(4 marks)**
- (b) A bike company has invested in a new sports bike assembling plant and it has been estimated that after assembling  $y$  sports bikes, the profit,  $p$  in Shs' 000' is given by the function  $P(y) = 4y^2 - 600y + 12,500$ .

**Required:**

- (i) Find the values of  $y$  for which the profit is zero. **(4 marks)**
- (ii) Determine the maximum profit. **(3 marks)**

- (c) A landlord in urban area owns three houses with one bedroom, two bedrooms and three bedrooms respectively. The total rent the landlord receives per month is Shs 1,240,000. The landlord needs to make mini repairs on the houses and it costs 10% of the one bedroomed house rent, 20% of the two bedroomed house rent and 30% of the three bedroomed house rent. The total bill for repairs is Shs 276,000. Given that the three bedroomed house rent is twice that of one bedroomed house;

**Required:**

Compute the rent for each of the three houses.

**(9marks)****(Total 20 marks)****Question 5**

- (a) Define the following terms as applied in linear programming:
- (i) 'Feasible region'. **(1 mark)**
  - (ii) 'Optimum solution'. **(1 mark)**
- (b) A factory manufacturing floor tiles is to use three ingredients A, B and C in varying proportions to produce three new brands of tiles: Oglas (O), Puncher (P) and Gaura (G). The factory wishes to produce at least 30, 40 and 50 units of O, P and G respectively in the initial phase. Each ton of A yields 10 units of O, 20 units of P and 30 units of G, while each ton of B yields 20 units of O, 10 units of P and 10 units of G and a ton of C yields 30 units of O, 20 units of P and 10 units of G.

Given that the factory used  $x_1$  tons of A  $x_2$  tons of B  $x_3$  tons of C and that the cost of each ton of ingredient A, B and C is Shs 10 million, Shs 12 million and Shs 15 million shillings respectively;

**Required:**

- (i) Formulate a linear programming model for the above information. **(2 marks)**
  - (ii) Generate the dual to the linear programming model in (b) (i) above. **(2 marks)**
  - (iii) Using a simplex tableau method solve the dual in (b) (ii) above to determine the minimum cost of production. **(7 marks)**
- (c) Marks' Service Garage is very popular with the latest vehicle models in town. The Supervisor, Ms. Mariam Akangas is particularly interested in the fuel consumption of the vehicles they handle. She has compiled the following data comparing the age of the vehicles (in years) and the distance per litre of fuel (in kilometres, km):

Age ( $x$ )	2	4	6	8	10	12	14
Distance ( $y$ )	18.0	17.9	17.5	16.8	16.6	16.2	16.0

**Required:**

- (i) Determine the regression equation of  $x$  on  $y$  by least squares method.

**(6 marks)**

- (ii) Find the age of a car that covers 17.2 km per litre of fuel.

**(1 mark)****(Total 20 marks)****Question 6**

- (a) Distinguish between the following concepts in relation to network analysis:
- (ii) 'Total float' and 'free float'. **(2 marks)**
  - (iii) 'Normal cost' and 'crash cost'. **(2 marks)**
- (b) (i) Distinguish between maximax and maximin criterion as applied in decision theory.

**(2 marks)**

- (ii) Kato is planning to construct a building in Katwe Division in Masaka Municipality. He carried out research on the estimated profits (in Shs 'million') depending on the proposal to elevate the municipality to city status. The table below shows the estimated profits for the proposed use of the building once completed.

	State of nature	
	Not elevated to city status	Elevated to city status
Alternative		
Apartments	8	8
Hotel	5	15
Shopping mall	-11	22
Probability	0.4	0.6

**Required:**

Calculate the expected profit for each alternative using the expected monetary value approach and advise Kato on the best alternative to choose.

**(4 marks)**

- (c) Ringa Engineering Ltd has decided to undertake some research to create a selling strategy for their new product. The company operations manager has identified the activities A to F and their associated duration time (in

weeks), and cost (in Shs '000') and schedule of implementation as given in the table below:

Task	Pre-task	Normal time	Crash time	Normal cost	Crash cost
A	None	4	3	360	460
B	None	8	5	300	510
C	A	5	3	170	270
D	A	9	7	220	300
E	B, C	5	3	200	360
F	D, E	2	2	200	250

**Required:**

- (i) Draw an activity on node network and determine critical activities. **(4 marks)**
- (ii) Determine the cost slope for critical activities. **(3 marks)**
- (iii) Determine the cost of the project if activities C, E and F are crashed by 2, 2 and 1 week(s) respectively.

**(3 marks)**

**(Total 20 marks)**

**FORMULAE**

1. Combination  ${}^nC_r = \frac{n!}{(n-r)!r!}$
2. Permutations  ${}^nP_r = \frac{n!}{(n-r)!}$
3. Mean of the binomial distribution =  $np$
4. Standard deviation =  $\sqrt{npq}$
5. Variance of the binomial distribution =  $np(1-p)$
6. Standard error of population proportion  $S_{ps} = \sqrt{\frac{pq}{n}}$
7. Spearman's rank correlation coefficient  $r = 1 - \frac{6\sum d^2}{n(n^2-1)}$
8. Product moment coefficient of correlation = 
$$\frac{n\sum xy - \sum x \sum y}{\sqrt{(n\sum x^2 - (\sum x)^2) \times (n\sum y^2 - (\sum y)^2)}}$$
9. Cost slope = 
$$\frac{\text{crash cost} - \text{normal cost}}{\text{normal time} - \text{crash time}}$$
10. Harmonic mean (ungrouped data)  $hm = \frac{n}{\sum \frac{1}{x}}$
11. Sample mean  $\bar{x} = \frac{\sum x}{n}$
12. Harmonic mean (grouped data)  $hm = \frac{n}{\sum \frac{f}{x}}$
13. Quartile coefficient of dispersion =  $\frac{Q_3 - Q_1}{Q_3 + Q_1}$
14. Bowley's coefficient of skewness =  $\frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$
15. Mean  $\bar{x} = A + \frac{\sum fd}{\sum f}$  or Mean  $\bar{x} = \frac{\sum fx}{\sum f}$



16. Median  $= Lb + \left( \frac{\frac{N}{2} - Cfb}{fm} \right) C$
17. Mode  $= lm + \left( \frac{d_1}{d_1 + d_2} \right) C$
18. Variance  $Var(x) = \frac{\sum fx^2}{\sum f} - \left( \frac{\sum fx}{\sum f} \right)^2$
19. Standard deviation  $\delta = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2} = \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}}$
20. Sample standard deviation  $s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$
21. Least squares regression equation of y on x is given by;  $y = a + bx$   
Where;  $b = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2}$  and  $a = \frac{\sum y}{n} - b \frac{\sum x}{n}$
22. Least squares regression equation of x on y is given by;  $x = c + dy$   
Where  $c = \frac{\sum x}{n} - d \frac{\sum y}{n}$  and  $d = \frac{n \sum xy - \sum x \sum y}{n \sum y^2 - (\sum y)^2}$
23. Standardising normal.  $z = \frac{\bar{x} - \mu}{\sigma}$
24. Confidence interval for sample mean  $= \bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$
25.  $\chi^2 = \sum \frac{(O - E)^2}{E}$
26. Confidence interval of proportion  $= p \pm z_{\alpha/2} \sqrt{\frac{pq}{n}}$
27. Pearson coefficient of skewness  $Sk = \frac{(\bar{x} - \text{mode})}{s_d}$  or  $Sk = \frac{3(\bar{x} - \text{median})}{s_d}$
28. Expectation  $= \sum xP(X = x)$
29. Laspeyres' price index  $= \frac{\sum (p_1 \times q_0)}{\sum (q_0 \times p_0)} \times 100$

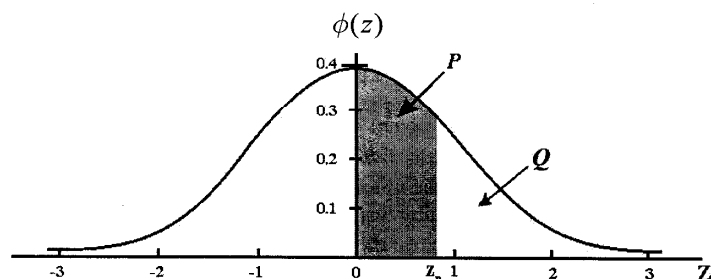
30. Paasche's Model  $= \frac{\sum (p_1 \times q_1)}{\sum (q_1 \times p_0)} \times 100$
31. Weighted aggregate price index  $= \frac{\sum wv_n}{\sum wv_0} \times 100$
32. Additive law of probability;  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
33. Conditional probability  $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$
34. Independence of A, B  $P\left(\frac{A}{B}\right) = P(A)$  or  $P(A \cap B) = P(A) \times P(B)$
35. Continuous compounding  $A = P(1+r)^n + \frac{b(1+r)^n - b}{r}$
36. Quotient rule of differentiation  $f = \frac{vu^1 - uv^1}{v^2}$ ; where  $f = \frac{u}{v}$
37. Poisson Model  $P(X = x) = e^{-\lambda} \frac{\lambda^x}{x!}$

CUMULATIVE NORMAL DISTRIBUTION $P(z)$											ADD								
Z	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	0.0000	0040	0080	0120	0160	0199	0239	0279	0319	0359	4	8	12	16	20	24	28	32	36
0.1	0.0398	0438	0478	0517	0557	0596	0636	0675	0714	0753	4	8	12	16	20	24	28	32	36
0.2	0.0793	0832	0871	0910	0948	0987	1026	1064	1103	1141	4	8	12	15	19	22	27	31	35
0.3	0.1179	1217	1255	1293	1331	1368	1406	1443	1480	1517	4	8	11	15	19	22	26	30	34
0.4	0.1554	1591	1628	1664	1700	1736	1772	1808	1844	1879	4	7	11	14	18	22	25	29	32
0.5	0.1915	1950	1985	2019	2054	2088	2123	2157	2190	2224	3	7	10	14	17	21	24	27	31
0.6	0.2257	2291	2324	2357	2389	2422	2454	2486	2517	2549	3	6	10	13	16	19	23	26	29
0.7	0.2580	2611	2642	2673	2704	2734	2764	2794	2823	2852	3	6	9	12	15	18	21	24	27
0.8	0.2881	2910	2939	2967	2995	3023	3051	3078	3106	3133	3	6	8	11	14	17	20	22	25
0.9	0.3159	3186	3212	3238	3264	3289	3315	3340	3365	3389	3	5	8	11	13	16	19	22	24
1.0	0.3413	3438	3461	3485	3508	3531	3554	3577	3599	3621	3	5	7	10	12	15	17	20	22
1.1	0.3643	3665	3686	3708	3729	3749	3770	3790	3810	3830	2	4	7	9	11	13	15	18	20
1.2	0.3849	3869	3888	3907	3925	3944	3962	3980	3997	4015	2	4	6	8	10	12	14	16	18
1.3	0.4032	4049	4066	4082	4099	4115	4131	4147	4162	4177	2	4	6	8	10	11	13	15	17
1.4	0.4192	4207	4222	4236	4251	4265	4279	4292	4306	4319	2	3	5	6	8	10	11	13	14
1.5	0.4332	4345	4357	4370	4382	4394	4406	4418	4429	4441	1	3	4	5	7	9	11	13	14
1.6	0.4452	4463	4474	4484	4495	4505	4515	4525	4535	4545	1	3	4	5	7	9	11	13	14
1.7	0.4554	4564	4573	4582	4591	4599	4608	4616	4625	4633	1	2	3	4	5	7	9	11	12
1.8	0.4641	4649	4656	4664	4671	4678	4686	4693	4699	4706	1	1	2	3	4	5	7	9	10
1.9	0.4713	4719	4726	4732	4738	4744	4750	4756	4761	4767	1	1	2	2	3	4	5	7	8
2.0	0.4772	4778	4783	4788	4793	4798	4803	4808	4812	4817	0	1	1	2	2	3	3	4	4
2.1	0.4821	4826	4830	4834	4838	4842	4846	4850	4854	4857	0	1	1	2	2	2	3	3	4
2.2	0.4861	4864	4868	4871	4875	4878	4881	4884	4887	4890	0	1	1	1	2	2	2	3	3
2.3	0.4893	4896	4898	4901	4904	4906	4909	4911	4913	4916	0	0	1	1	1	2	2	2	2
2.4	0.4918	4920	4922	4925	4927	4929	4931	4932	4934	4936	0	0	1	1	1	1	1	2	2
2.5	0.4938	4940	4941	4943	4945	4946	4948	4949	4951	4952									
2.6	0.4953	4955	4956	4957	4959	4960	4961	4962	4963	4964									
2.7	0.4965	4966	4967	4968	4969	4970	4971	4972	4973	4974									
2.8	0.4974	4975	4976	4977	4977	4978	4979	4979	4980	4981									
2.9	0.4981	4982	4982	4983	4984	4984	4985	4985	4986	4986									
3.0	0.4987	4990	4993	4995	4997	4998	4998	4999	4999	5000									

The table gives  $P(z) = \int_0^z \phi(z) dz$

If the random variable  $Z$  is distributed as the standard normal distribution  $N(0,1)$  then:

1.  $P(0 < Z < z_p) = P(\text{Shaded Area})$
2.  $P(Z > Z_p) = Q = \frac{1}{2} - P$
3.  $P(Z > |Z_p|) = 1 - 2P = 2Q$



**PERCENTAGE POINTS OF THE CHI-SQUARE ( $\chi^2$ ) DISTRIBUTION  $\chi^2_Q$** 

Probability Q										
$\nu$	0.995	0.990	0.975	0.950	0.100	0.050	0.025	0.010	0.005	0.001
1	0.0 <sup>4</sup> 393	0.0 <sup>3</sup> 157	0.0 <sup>3</sup> 982	0.0 <sup>2</sup> 393	2.706	3.841	5.024	6.635	7.879	10.83
2	0.0100	0.0201	0.0506	0.1026	4.605	5.991	7.378	9.210	10.60	13.82
3	0.0717	0.1148	0.2158	0.3518	6.251	7.815	9.348	11.34	12.84	16.27
4	0.2070	0.2971	0.4844	0.7107	7.779	9.488	11.14	13.28	14.86	18.47
5	0.4117	0.5543	0.8312	1.145	9.236	11.07	12.83	15.09	16.75	20.52
6	0.6757	0.8721	1.237	1.635	10.64	12.59	14.45	16.81	18.55	22.46
7	0.9893	1.239	1.690	2.167	12.02	14.07	16.01	18.48	20.28	24.32
8	1.344	1.646	2.180	2.733	13.36	15.51	17.53	20.09	21.95	26.12
9	1.735	2.088	2.700	3.325	14.68	16.92	19.02	21.67	23.59	27.88
10	2.156	2.558	3.247	3.940	15.99	18.31	20.48	23.21	25.19	29.59
11	2.603	3.053	3.816	4.575	17.28	19.68	21.92	24.73	26.76	31.26
12	3.074	3.571	4.404	5.226	18.55	21.03	23.34	26.22	28.30	32.91
13	3.565	4.107	5.009	5.892	19.81	22.36	24.74	27.69	29.82	34.53
14	4.075	4.660	5.629	6.571	21.06	23.68	26.12	29.14	31.32	36.12
15	4.601	5.229	6.262	7.261	22.31	25.00	27.49	30.58	32.80	37.70
16	5.142	5.812	6.908	7.962	23.54	26.30	28.85	32.00	34.27	39.25
17	5.697	6.408	7.564	8.672	24.77	27.59	30.19	33.41	35.72	40.79
18	6.265	7.015	8.231	9.390	25.99	28.87	31.53	34.81	37.16	42.31
19	6.844	7.633	8.907	10.12	27.20	30.14	32.85	36.19	38.58	43.82
20	7.434	8.260	9.591	10.85	28.41	31.41	34.17	37.57	40.00	45.31
21	8.034	8.897	10.28	11.59	29.62	32.67	35.48	38.93	41.40	46.80
22	8.643	9.542	10.98	12.34	30.81	33.92	36.78	40.29	42.80	48.27
23	9.260	10.20	11.69	13.09	32.01	35.17	38.08	41.64	44.18	49.73
24	9.886	10.86	12.40	13.85	33.20	36.42	39.36	42.98	45.56	51.18
25	10.52	11.52	13.12	14.61	34.38	37.65	40.65	44.31	46.93	52.62
26	11.16	12.20	13.84	15.38	35.56	38.89	41.92	45.64	48.29	54.05
27	11.81	12.88	14.57	16.15	36.74	40.11	43.19	46.96	49.64	55.48
28	12.46	13.56	15.31	16.93	37.92	41.34	44.46	48.28	50.99	56.89
29	13.12	14.26	16.05	17.71	39.09	42.56	45.72	49.59	52.34	58.30
30	13.79	14.95	16.79	18.49	40.26	43.77	46.98	50.89	53.67	59.70
40	20.71	22.16	24.43	26.51	51.81	55.76	59.34	63.69	66.77	73.40
50	27.99	29.71	32.36	34.76	63.17	67.50	71.42	76.15	79.49	86.66
60	35.53	37.48	40.48	43.19	74.40	79.08	83.30	88.38	91.95	99.61
70	43.28	45.44	48.76	51.74	85.53	90.53	95.02	100.4	104.2	112.3
80	51.17	53.54	57.15	60.39	96.58	101.9	106.6	112.3	116.3	124.8
90	59.20	61.75	65.65	69.13	107.6	113.1	118.1	124.1	128.3	137.2
100	67.33	70.06	74.22	77.93	118.5	124.3	129.6	135.8	140.2	149.4

The function tabulated is  $\chi^2_Q$  defined by

$$\int_{\chi^2_Q}^{\infty} f(x) dx = Q; \quad f(x) = \frac{1}{2^{1/2}(\frac{1}{2}\nu - 1)!} x^{1/2-1} e^{-x/2} (x > 0)$$

where  $f(x)$  is the probability density of the  $\chi^2$  distribution for  $\nu$  degrees of freedom.

Interpolation  $\nu$ -wise for  $\nu > 30$  gives adequate values (but errors up to 5 units in the last figure may occur for the smaller  $\nu$ ). For  $\nu > 100$  the distribution of  $\sqrt{2\chi^2}$  is approximately normal with mean  $\sqrt{2(\nu - 1)}$  and unit variance.

Note:  $0.0^4_2 = 0.00002$   
 $0.0^3_3 = 0.0003$   
 $0.0^2_4 = 0.004$

PERCENTAGE POINTS OF STUDENT'S  $t$ -DISTRIBUTION  $t_Q$ 

$\nu$	Probability*									$Q$ $2Q$
	0.25 0.50	0.10 0.20	0.05 0.10	0.025 0.050	0.01 0.02	0.005 0.010	0.0025 0.0050	0.001 0.002	0.0005 0.0010	
1	1.000	3.078	6.314	12.71	31.82	63.66	127.3	318.3	636.6	
2	0.816	1.886	2.920	4.303	6.965	9.925	14.09	22.33	31.60	
3	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.21	12.92	
4	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610	
5	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869	
6	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959	
7	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408	
8	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041	
9	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781	
10	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587	
11	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437	
12	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318	
13	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221	
14	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140	
15	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073	
16	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015	
17	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965	
18	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922	
19	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883	
20	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850	
21	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819	
22	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792	
23	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767	
24	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745	
25	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725	
26	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707	
27	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690	
28	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674	
29	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659	
30	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646	
40	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551	
60	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460	
120	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373	
$\infty$	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291	

The function tabulated is  $t_Q$  defined by

$$\int_{t_Q}^{\infty} f(t) dt = Q; \quad f(t) = \frac{(\frac{1}{2}\nu - \frac{1}{2})!}{\sqrt{(\nu\pi)(\frac{1}{2}\nu - 1)!}} \cdot \frac{1}{(1 + t^2/\nu)^{(\nu+1)/2}}$$

where  $f(t)$  is the probability density of the  $t$ -distribution.

Interpolation  $\nu$ -wise should be linear in  $120/\nu$  for  $\nu > 30$ .

Use (i) upper row for one tail-tests

(i) lower row for two tail-tests

If  $x$  is a random variable with the  $t$ -probability distribution for  $\nu$  degrees of freedom, the probability that  $x > t_Q$  is  $Q$  and the probability that  $|x| > t_Q$  is  $2Q$ .

The graph shows the form of the distribution for  $\nu = 2$ . The shaded area represents the probability  $Q$ . For large  $\nu$  the distribution approximates to the normal distribution  $N(0,1)$ , shown by the dotted line.

